Longmans' Mathematics

A five-year course

A graded series to cover the requirements of 'O' level in the fifth year (Stage Five). Stage Four reaches the standard required for 'other than "O" level' examinations.

STAGE ONE

Longmans' Mathematics

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Introduction

The authors of these books have written them with two main ideas in mind. The first is that mathematics is indivisible; it contains all the elements we know as arithmetic, algebra, geometry and trigonometry and each of these branches of the subject is dependent on all the others. For this reason we believe that all secondary school pupils should be taught *mathematics*. Furthermore, we believe that most secondary school pupils, even those less academically inclined, can learn mathematics provided that the emphasis is placed on the results and uses of the subject rather than the abstract reasoning behind the various processes. We have, therefore, concentrated on the application of mathematics to the solution of many practical problems in order to show its value to the pupils, and we have endeavoured to avoid the traditional 'academic' approach to the teaching of this subject.

Our second idea is this. We do not believe that pupils in secondary schools are particularly interested in the methods of teaching mathematics. They require a text-book to direct them to perform certain operations with clearly set out diagrams and worked examples for reference. A large number of graded questions is obviously necessary. On the other hand, we firmly believe that many teachers of mathematics will welcome a book which gives well-tried methods of teaching the various aspects of the subject, including some parts with which they are not so familiar or which their pupils have not attempted before. Separate answer books will be provided and we intend to include 'notes for teachers' in them.

Stages 1, 2, 3, 4 each represent one year's work for the pupil and Stage 4 covers all the required topics likely to be included in the Royal Society of Arts Examinations or other examinations within the terms of the Beloe Report. Stage 5 is specially designed for G.C.E. 'O' level candidates.

The books are designed for pupils in all secondary schools, particularly for those who appreciate the non-traditional approach to mathematics. If we succeed in widening the opportunities of learning this most fascinating of subjects to many boys and girls then we shall feel our efforts have been well rewarded.

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1 Solid Geometry

As science and engineering have developed, shapes have become more and more complicated. In this chapter you are going to start with a study of some of the simpler shapes.

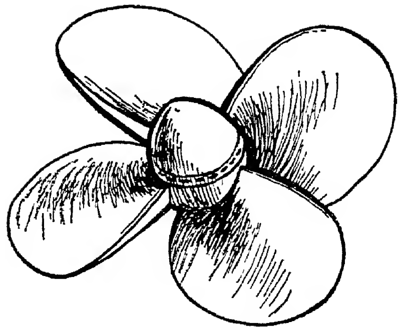


fig. 1.1

1 The Cuboid

'Cuboid' is the mathematical name for a box. It is the commonest solid and is used for a great variety of things, from containers to houses. Collect pictures or draw six objects which are cuboids for homework.

Look at the sketch of a cuboid or at your teacher's model.

How many sides has it?
How many edges? How many corners?
What shapes are its sides?
Are the sides all the same?
What can we say about opposite sides?

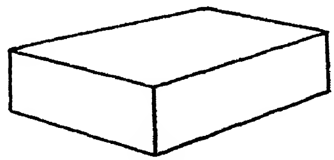


fig. 1.2

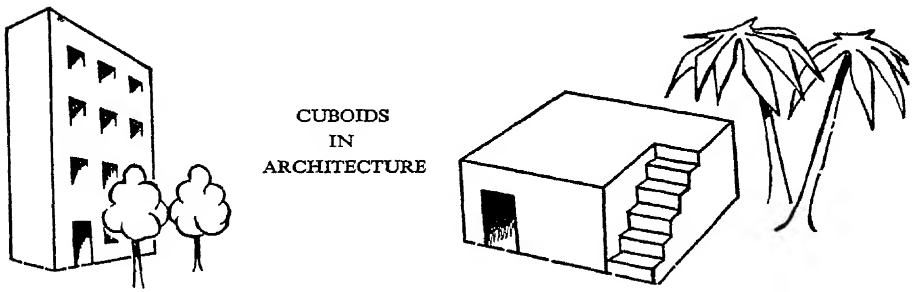


fig. 1.3

2 The Cube

A cuboid which has all its sides the same length is known as a CUBE. Look carefully at your teacher's model.

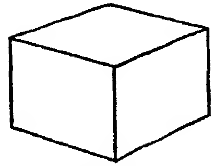


fig. 1 4

What shape are the faces of the cube?
Are the faces all the same?
What can you say about the lengths of the edges?
Suppose the cube is hollow; cutting along all the edges except the four edges of the base, and flattening it out we obtain the shape in fig. 1.5.

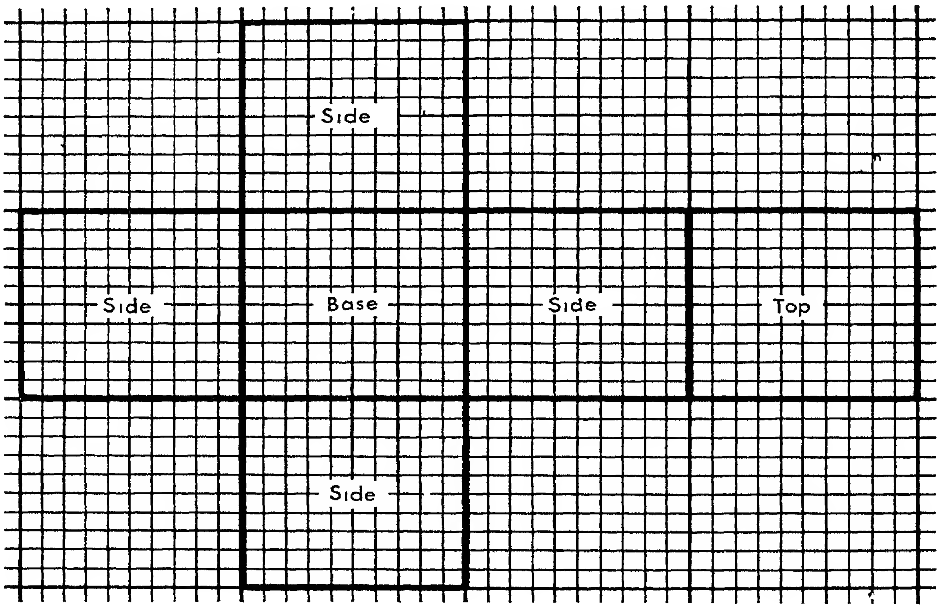


fig. 1.5

This is known as the 'net' of the cube. The nets of some shapes are very difficult to obtain and need a good deal of calculation. Consider for example the shape needed to form the body of a car.

How many small squares cover one side of the cube?
How many small squares cover the whole of the cube's faces?

Draw out the net of a cube with each edge 2 inches long. Cut out the pattern, fold along the lines and construct the cube by joining the edges with Sellotape.

How many cubes with edge 1 inch would fit into your cube?

ANY solid which has all its edges the same length, and all its faces the same shape, and all its angles the same size, is known as a 'regular' solid.

Is the cube a regular solid?
Is the cuboid a regular solid?

3 The Pyramid

Cubes, no matter what size, are all of one shape. Cuboids are also of one general shape, although differing in sizes. Pyramids, however, are not all the same shape. Figs. 1.6 shows a number of solids all of which are pyramids.

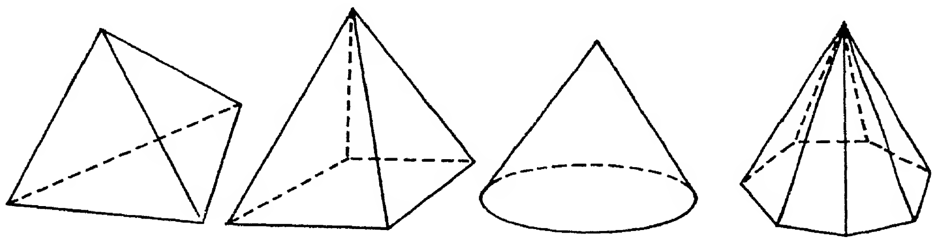


fig. 1.6

You will see many different shaped pyramids as church steeples. For homework see how many different shapes you can find.

What are the differences between the pyramids shown?
What makes all these shapes pyramids?

The pyramid used over 3,000 years ago by the Egyptians is the 'square' pyramid, fig. 1.7. Why is it called a 'square' pyramid?

How many faces has this pyramid?
How many corners? How many edges?
Are all the faces the same shape?
What shape are the sides?
What shape is the base?
Is it a regular solid?

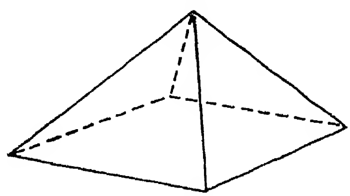


fig 1.7

Copy out the net as shown in fig. 1.8 on to squared paper, cut it out and stick it together to make the square pyramid.

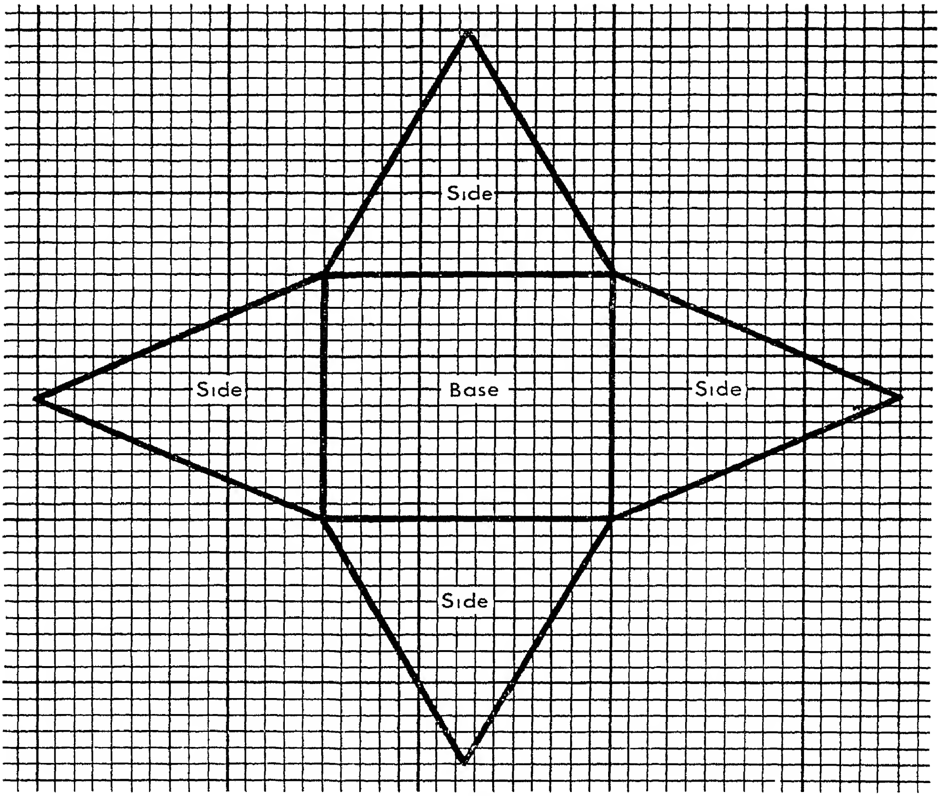


fig. 1 8

In this pyramid the top point, known as the apex, should be directly over the centre of the base. When this happens the pyramid is known as a 'right' pyramid. A pyramid which is not a right pyramid is shown in fig. 1.9. The net for this pyramid is shown in fig. 1.10 below. Copy out the net on to squared paper by counting the squares, fold it to form the pyramid and stick the edges together.

Take three of these pyramids and fit them together to form a cube.

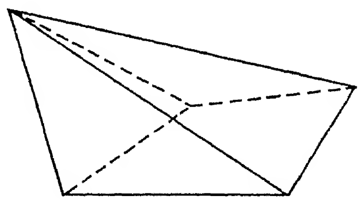


fig 1.9

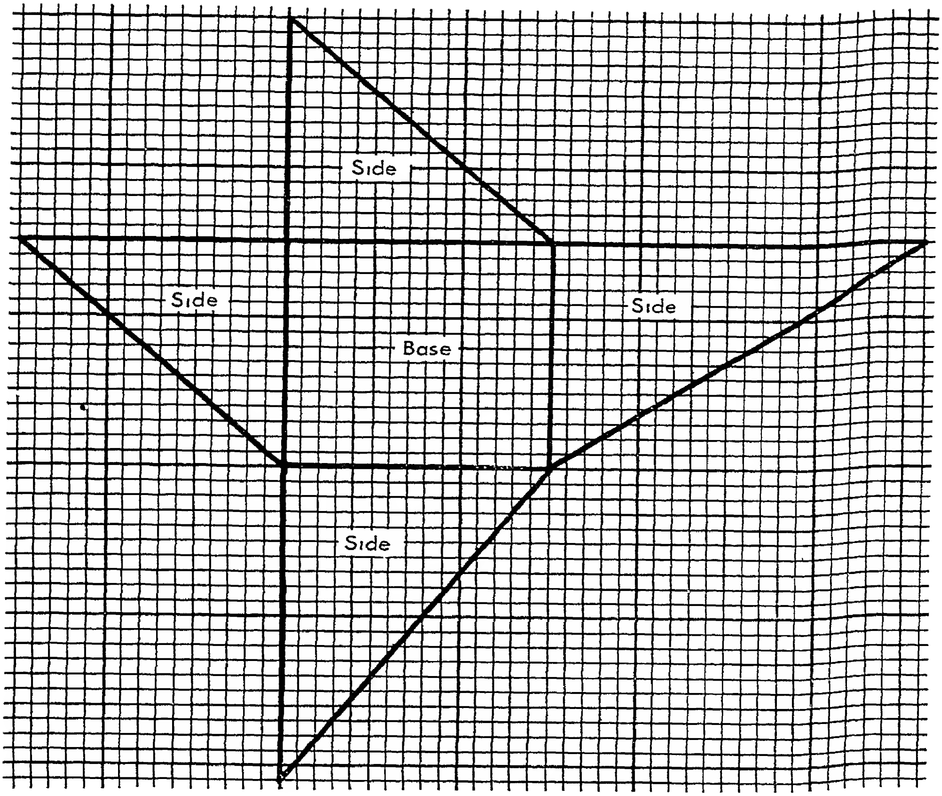


fig. 1.10

Notice that the bases of the pyramids become sides of the cube. What fraction of the cube is one pyramid?

4 The Prism

In the above section of work you met a set of shapes all of which were different but all of which were pyramids. Look at the set of shapes in fig. 1.11. All these shapes are PRISMS.

Can you see what makes a shape a prism?

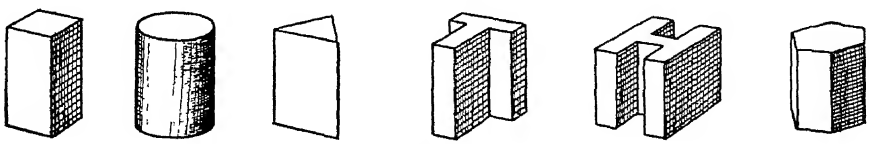


fig. 1.11

(a) THE SQUARE PRISM

Fig. 1.12 shows a square prism. Look at a model if your teacher has one.

How does it differ from a cube?
How does it differ from the cuboid in fig. 1.2?
Are all its sides the same shape?
What shape are the sides?
Is it a regular solid?

Draw out the net of the prism in fig. 1.12, cut it out, fold it and stick it to make the solid.

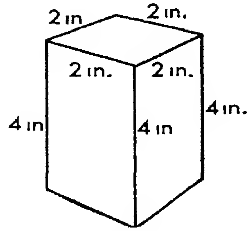


fig 1.12

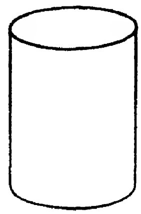


fig 1.13

(b) THE CIRCULAR PRISM OR CYLINDER

This is a common shape, used for example for tinned fruit. Cutting round the top and bottom and straight down one side and flattening out the tin would give the shape of the net as shown in fig. 1.14.

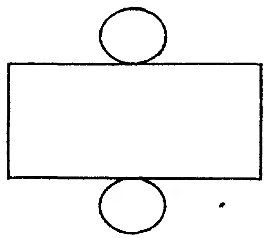


fig. 1.14

What shapes are there?
If you wished to measure the distance round the circle, how could you do this easily?

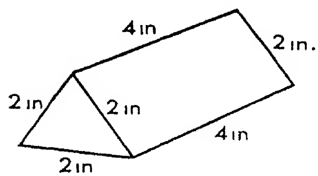


fig. 1.15

(c) THE TRIANGULAR PRISM

Draw out the net for the prism shown in fig. 1.15. What shape are the long sides?

A very clever mathematician, Leonard Euler, who lived in the eighteenth century, found that there was a connection between the number of faces, corners and edges of a solid. Copy out the table given opposite, count the faces, edges and corners of the solids suggested, fill in the table, and then see if you can find the connection.

	FACES	CORNERS	EDGES	FACES + CORNERS
Cube				
Cuboid				
Square Pyramid				
Triangular Pyramid				
Triangular Prism				
T-Shaped Prism				
H-Shaped Prism				

The last two need care—they are difficult.

Extra Work
Make up the solids shown here in stiff paper.

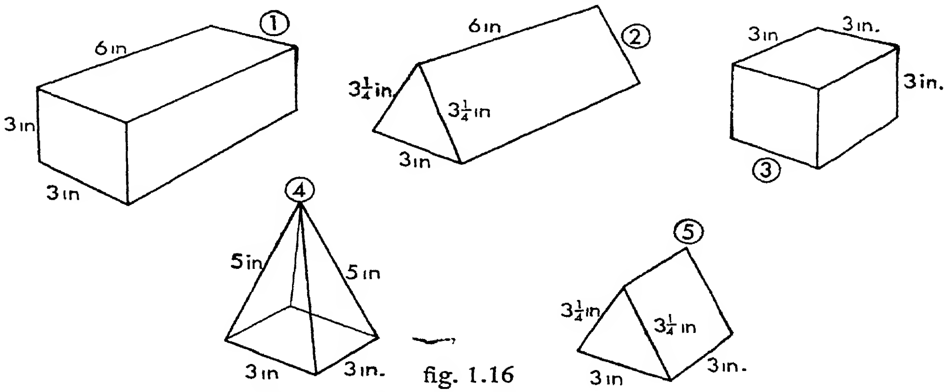
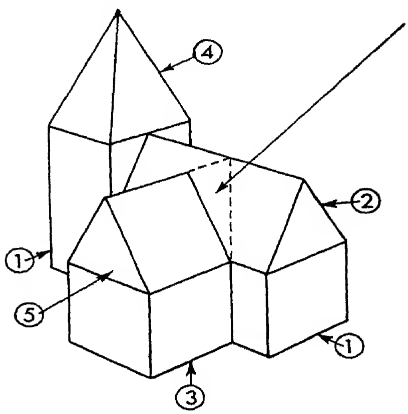


fig. 1.16

Assemble them to form a Norman Church.



You will need to make one piece twice, and there will be one piece short here. Can you make this piece?

fig. 1.17

2 Scale Drawing

Drawings have been called the 'language of the workshop'. When a builder builds a house, an engineer constructs a bridge, or a manufacturer makes a motor-car, drawings are made to show the various parts, how they fit with other parts and how they are to be constructed. By drawings, information can much more easily be passed on rather than by a long written description. This method is by no means new. Egyptian architects made drawings of the pyramids and temples of ancient Egypt, scratching them on clay tablets before those great monuments were actually built.

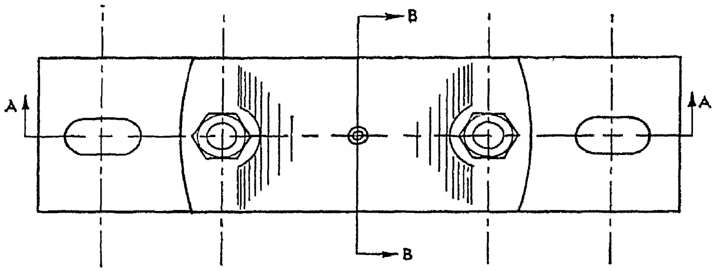


fig. 2.1. A modern scale drawing

Before making some drawings several points must be noted on the use of ruler and set-square.

Supposing you wish to draw a line joining two points, call them X and Y; the best way of doing this is to place the point of the pencil on the left-hand point, say X, slide the upper edge of the ruler gently up to it. When the ruler is touching the pencil turn the ruler so that the upper edge also passes through the second point Y. Press the ruler tightly to the paper and draw the line from left to right, fig. 2.2.

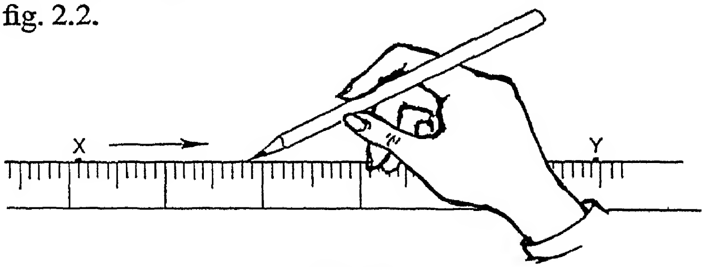


fig. 2.2

Exercise 2a

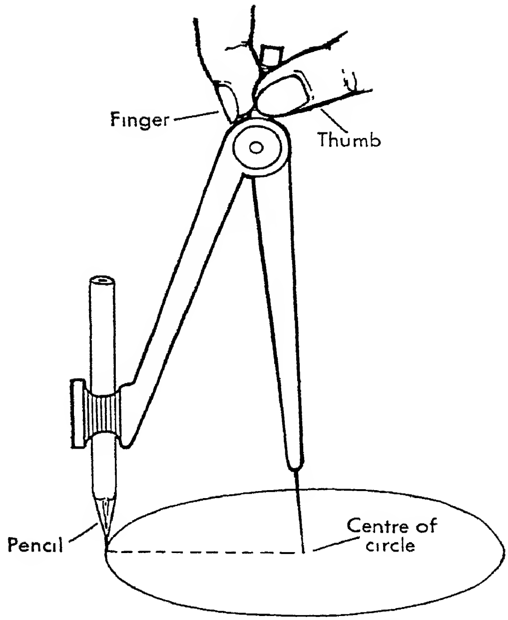


fig. 2.3. How to draw a circle

1. Draw the largest circle that you can in your exercise book. Make a small mark on the circumference of the circle, then place the compass point on this mark. With the compass open to the same distance as before, make another mark on the circumference. Place your compass point on this new mark and with the compass pencil 'step' out another mark.
Continue this round the circle and you should come back to the first mark. Join each of the points to its neighbours to form a six-sided figure (a hexagon).
Now join the first point to *all* the other points. Then move on and join the second point to *all* the other points; continue round joining *all* points to all other points. Do this carefully, drawing each line as described above. You will find that in the middle of the first hexagon you have drawn another six-sided figure, as in fig. 2.4.
Now very carefully join every corner of this figure to every other corner. What do you find in the centre now?

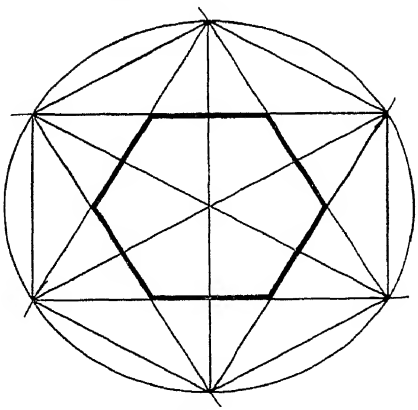


fig. 2.4

2. Draw two long lines OA, OB; mark off, with compasses, three equal lengths along OA: OL, LM, MN. They must each be the same and can be about 2 inches. The actual length does not matter. Along OB mark off three equal lengths, each about $1\frac{1}{4}$ inches, OC, CD, DE; again they must all be the same length but the actual size does not matter.
Now very carefully join LD, DN, CM and ME.

Let these lines cross at X and Y, as in fig. 2.5. Now, OXY should be in a straight line: test this by joining OY and see if this line passes through X.

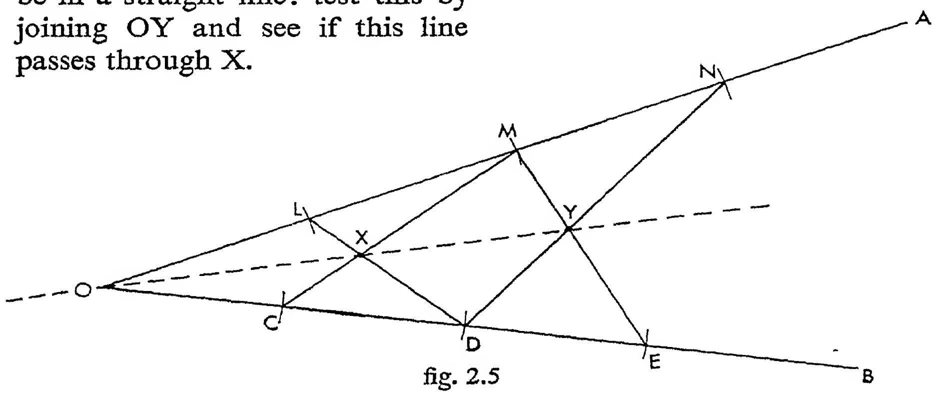


fig. 2.5

This is an interesting drawing, especially as the lengths we chose did not matter. This fact about OXY was discovered by a great mathematician called Pappus.

3. The following is a little more difficult. Copy fig. 2.6 as follows: start off as in (2) with two long lines OA, OB. Mark along OA three points at ANY distances, L, M and N. Mark along OB three points, again at *any* distances, C, D and E.

Join EM, ND and let them cross at F.
Join CM, LD and let them cross at G.
Join LE, NC and let them cross at H.

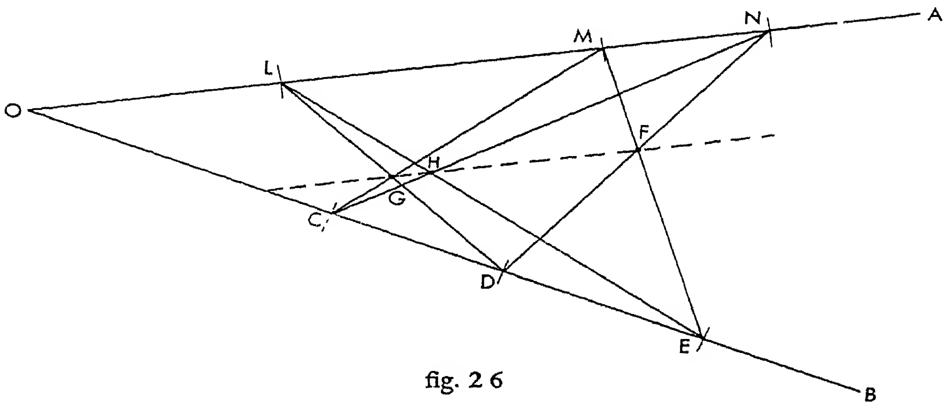


fig. 2 6

If you have done this accurately FGH will lie on a straight line. Test this.

Perpendicular Lines

One of the most important features in building or making things are 'square corners'. They must be perfectly accurate, otherwise, for example, walls would not stand upright and would tend to fall, and doors would not fit. So the next point in drawing is to construct a square corner.

When one line is square to another then the lines are said to be perpendicular. To draw one line perpendicular to another line you use an instrument called a 'set-square'. This enables you to set one line square to another.

Suppose you wish to draw a line perpendicular to the line XY at the point X, as in fig. 2.7.

fig. 2.7

First carefully place your ruler's upper edge along line XY, then lay the set-square carefully on this upper edge anywhere along the line, fig. 2.8. Next, slide the set-square along until you reach the point where you wish to draw the perpendicular. Hold the set-square firmly and draw the line away from yourself. Having drawn the line, mark-off the length required along it afterwards.

Parallel Lines

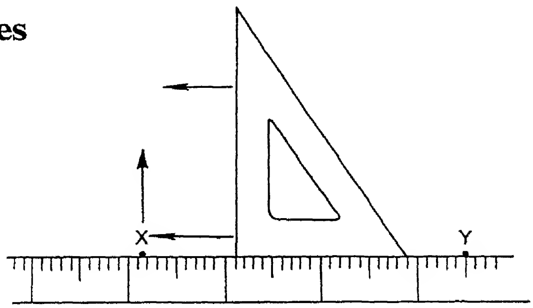


fig. 2.8

Using the set-square and ruler you can draw any number of lines by moving the set-square along, but keeping the ruler still, fig. 2.9.

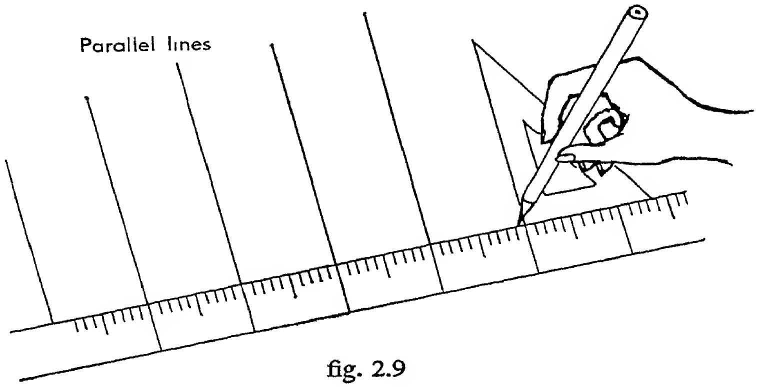


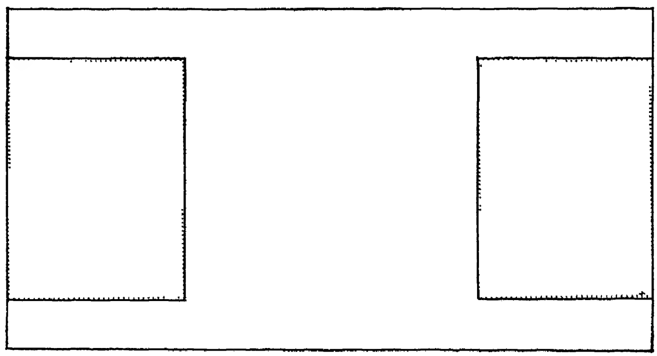
fig. 2.9

No matter how long these lines are made they will never cross each other. Such lines are called 'parallel' lines.

Exercise 2b

Copy the following drawings neatly into your book, first making a careful freehand sketch about quarter-size in the top right-hand corner. You will need only halves and quarters of an inch.

1.



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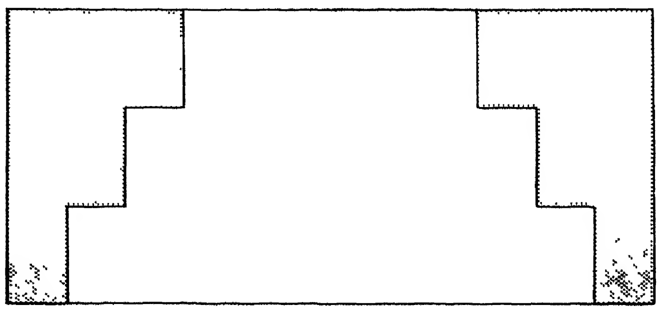

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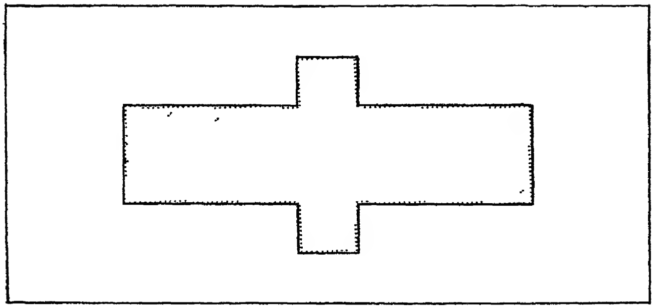

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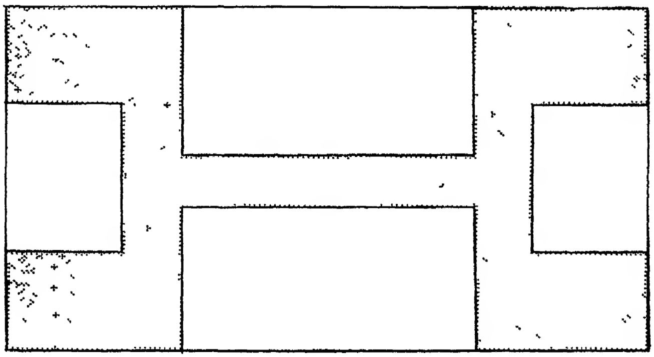

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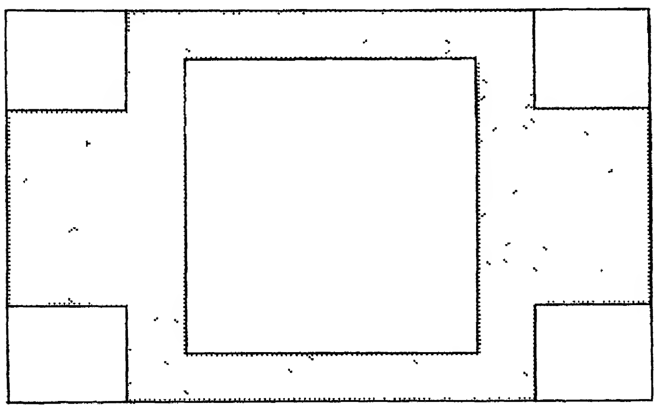

fig. 2.10

Exercise 2c

Copy the following drawings neatly into your book, first making a careful freehand sketch about quarter-size in the top right-hand corner of your paper. You will need sixteenths, eighths, quarters and halves of an inch.

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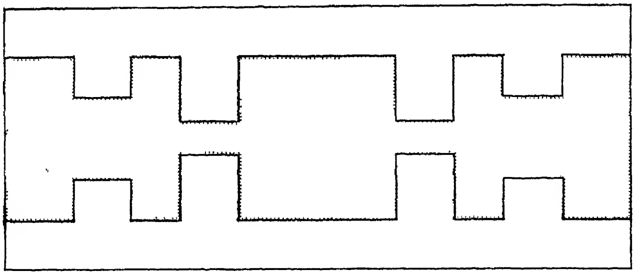

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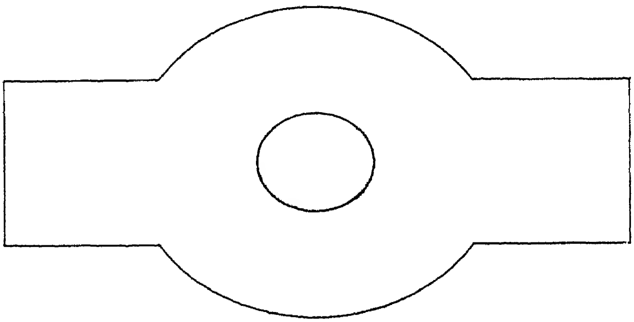

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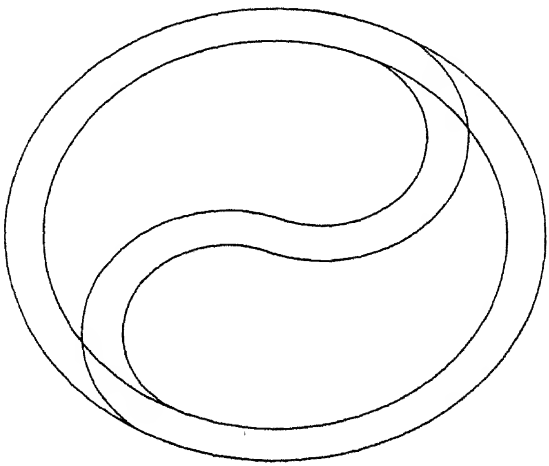

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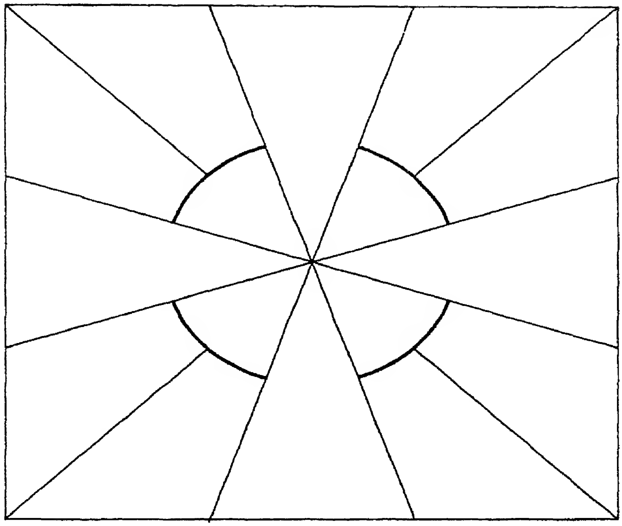

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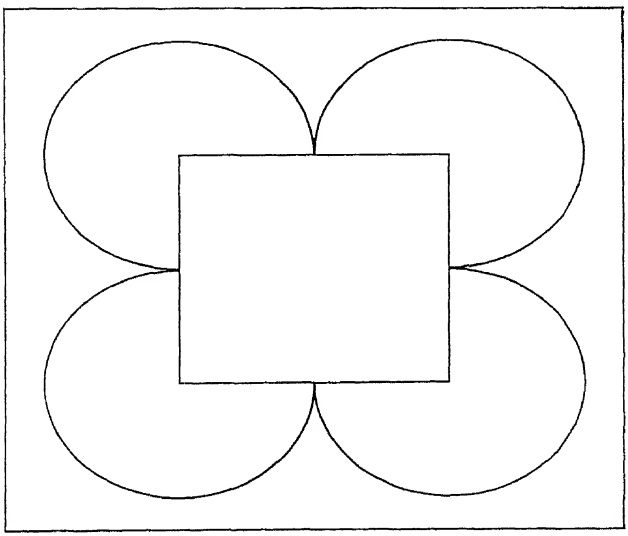


fig. 2.11

3 Plans and Elevations

The main purpose of making drawings is to give us a small picture on a flat (or *two-dimensional*) drawing paper, which we can easily carry about, of a large solid (or *three-dimensional*) object.
To see how this is done let us investigate how a solid rectangular prism resting on a flat level plane is represented in a two-dimensional diagram.

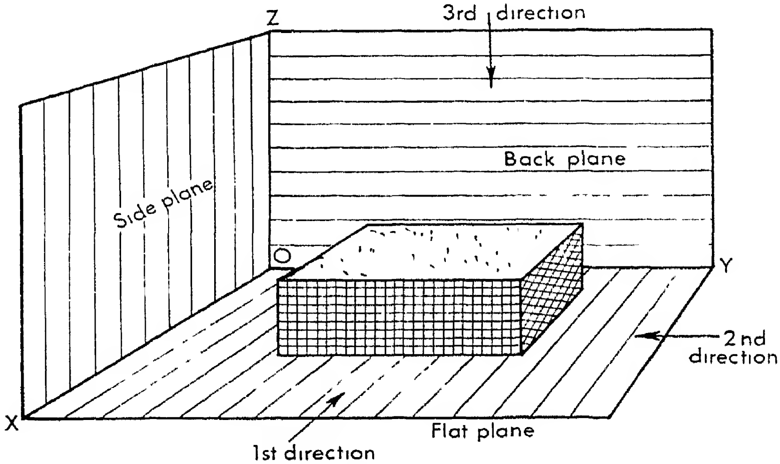


fig. 3.1

Fig. 3.1 shows the prism resting on the flat plane.

Look at the rectangular block from three directions. *First*, placing the eyes at the level of the flat plane, look in the direction of the arrow. The picture you see is the front face of the block, the rectangle shown in fig. 3.2. This appears to be drawn on the back plane.

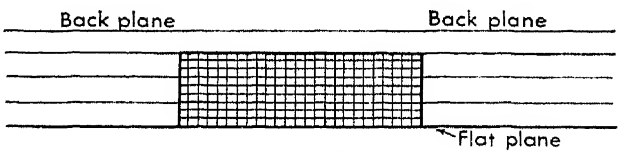


fig. 3.2

This picture is called the *front elevation.*

Second, keeping the eyes at the same level, move round to the end. The picture you see, the end face of the block which appears to be drawn on the side plane, is the rectangle in Fig. 3.3.

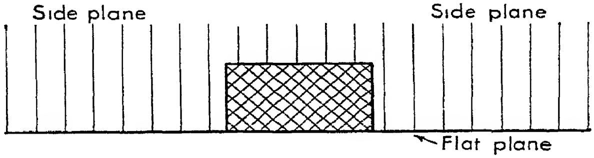


fig. 3.3

This picture we call the *end* or *side elevation.*

Finally, look at the object so that the eyes are directly above it. The picture you see is the top of the block, the rectangle shown in fig. 3.4 which appears to be drawn on the horizontal plane.

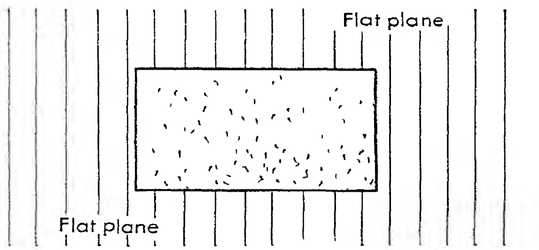


fig. 3 4

This picture we call the *plan.*

If the three planes are now flattened out, the three diagrams can be placed together and *joined by projection lines* so that the complete drawing will be shown thus:

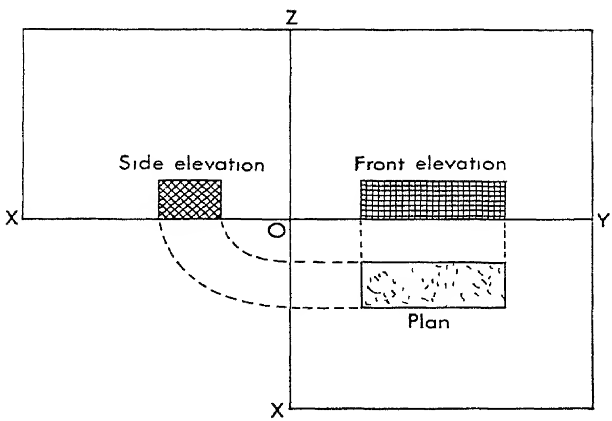


fig. 3.5

You are already familiar with *plans*; the maps you use in your atlas are plans of parts of the world as seen from an aeroplane. When your house was built the architect prepared a series of plans for the builders, to show how the ground-floor rooms and the upstairs rooms should be arranged. Aero-modellers and model-boat builders have used many 'blueprints', which are copies of plans drawn by the designer. From these examples you will notice that the drawings can be, and often are, considerably smaller than the object, and you should see from this a very important geometrical fact, namely, that two things —a drawing and a building—may be of quite different size but exactly the same shape. What is true about the shape of one is also true about the shape of the other.

All solid objects are built up of simple geometrical shapes, those most frequently used being rectangles, triangles and circles, and can therefore be represented diagrammatically by elevations and plans.

Exercise 3a

Copy these examples.

1. A 'lean-to' greenhouse (rectangles and triangles).

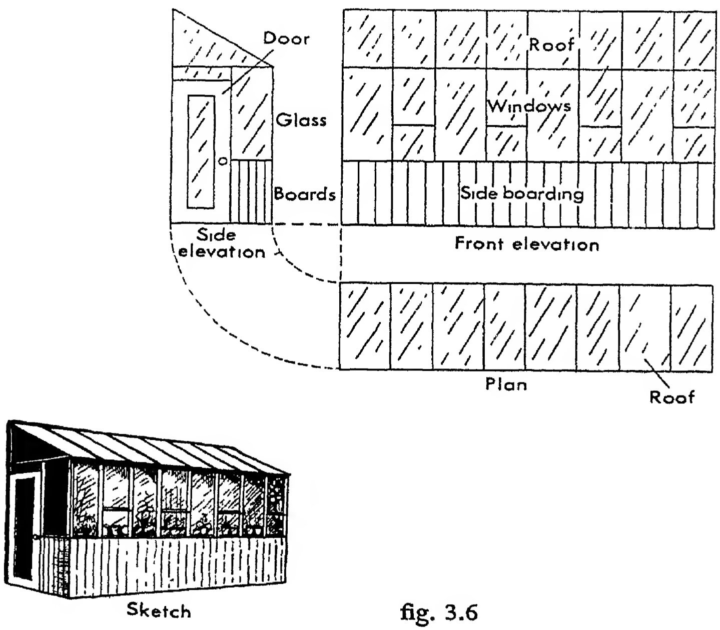


fig. 3.6

2. A one-bedroom chalet (rectangles and triangles).

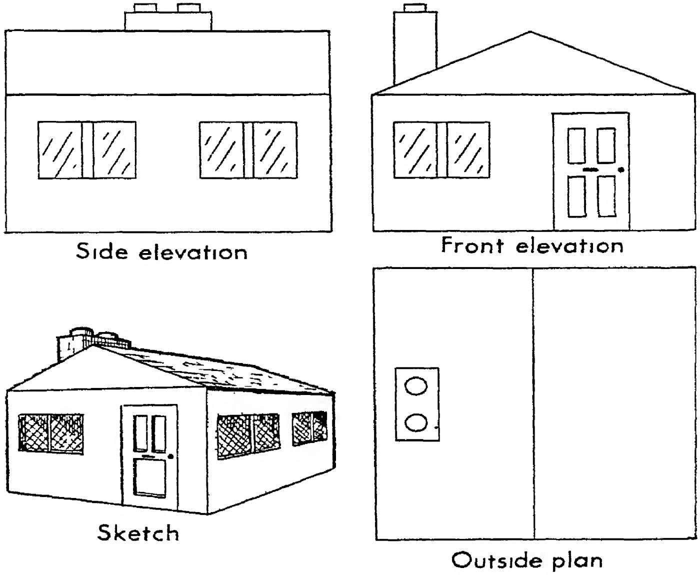


fig. 3.7

The builder will, however, require a plan of the rooms, so a further plan is drawn showing how the rooms are arranged. For the purpose of this we imagine the roof to be lifted off. A 'room plan' will then appear as follows:

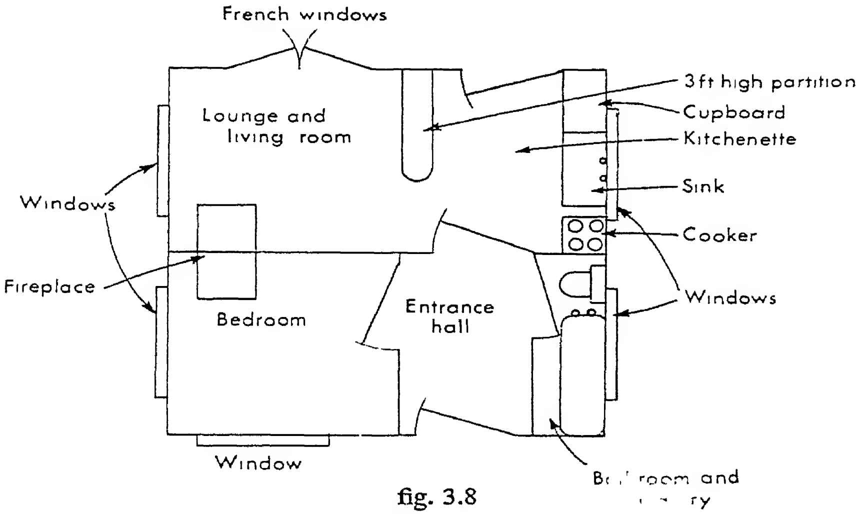


fig. 3.8

3. Shakespeare's Globe Theatre. (Simplified this is really a hollow hexagonal prism.)

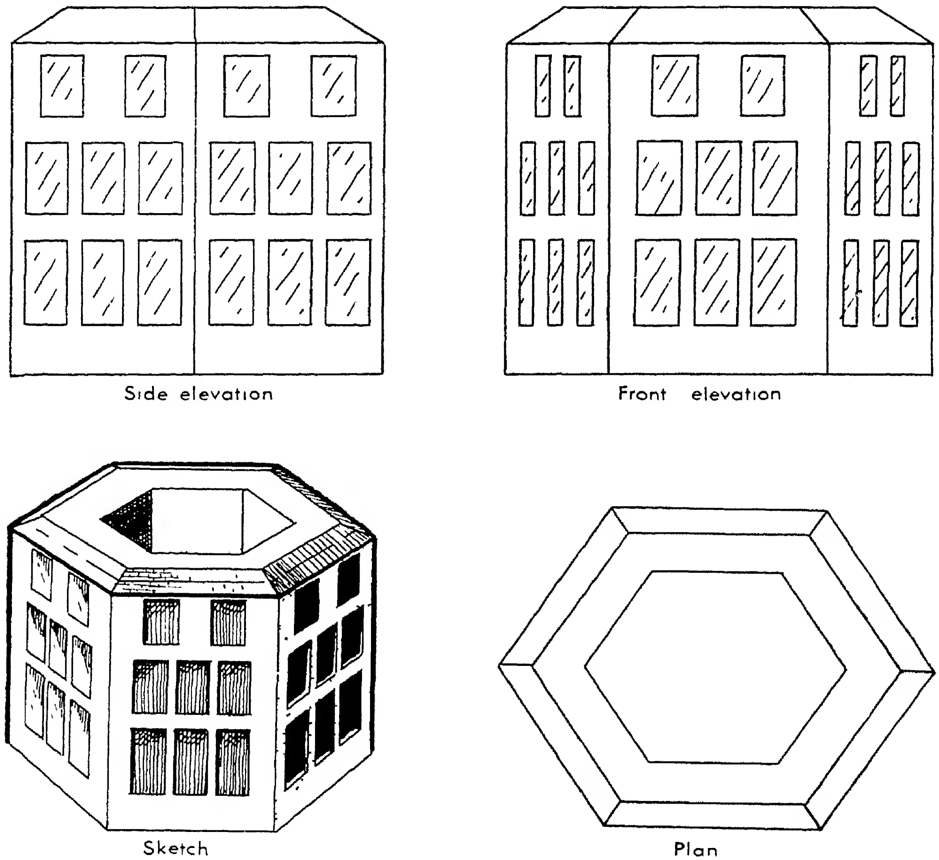


fig. 3.9

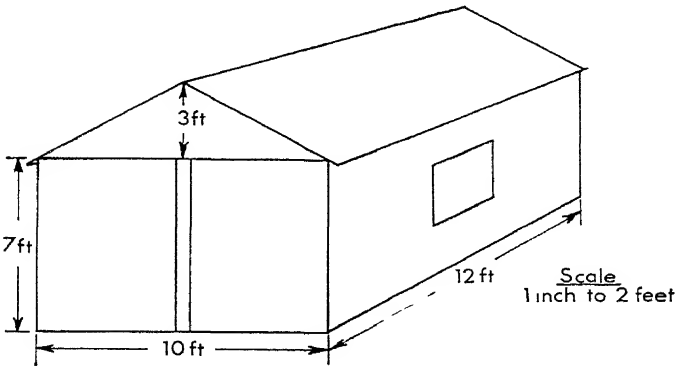
An engineer or an architect needs to be able to use his drawings in two ways. First, he must be able to produce the plan and elevations from any given object; and, second, he must be able to visualize what the object looks like when the drawings are provided. To help him do this he usually makes a *sketch* of the object, using the information (or data) given in the drawings. In some of the following exercises you will be required to do this.

Exercise 3b

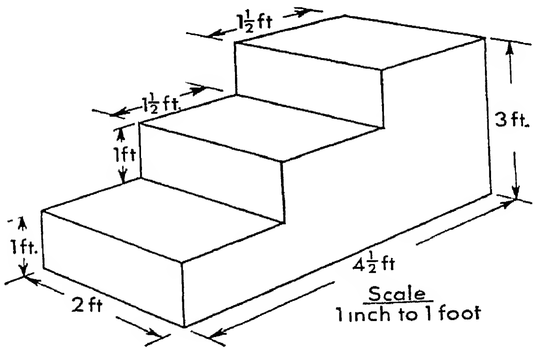
(All these exercises should be done on squared paper having one-inch squares with each side divided into tenths.)

1. A standard brick used in building is 9 in. long, $4\frac{1}{2}$ in. wide and 3 in. thick. If the brick is placed on a horizontal surface with a side 9 in. by $4\frac{1}{2}$ in. resting on the surface, draw the plan and elevations. *Scale* 1 : 2.

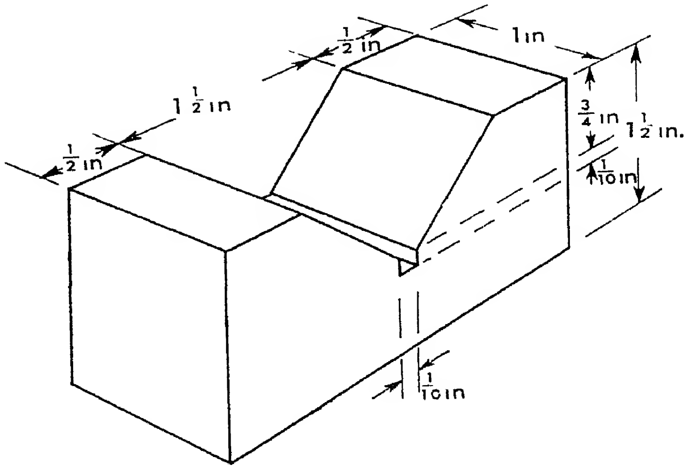
2. The diagram represents a garage.
Draw the plan and elevations of this diagram and of diagrams **3, 4** and **5**.



3.



4.



$\frac{1}{2}$ in
1 in
$1\frac{1}{2}$ in
$\frac{1}{2}$ in
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$1\frac{1}{2}$ in.
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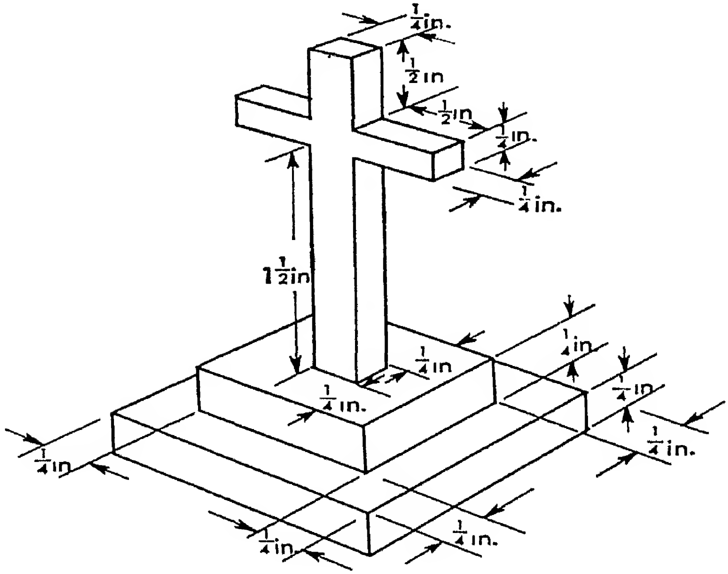
$\frac{1}{10}$ in

5.



$\frac{1}{4}$ in.
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fig. 3.10

4 Numbers

Magic Squares

Here is a square divided into 9 smaller squares. In each square one of the numbers from 1 to 9 is written.

4	3	8
9	5	1
2	7	6

Add across:

$4 + 3 + 8 = 15$
$9 + 5 + 1 = 15$
$2 + 7 + 6 = 15$

Add down:

$4 + 9 + 2 = 15$
$3 + 5 + 7 = 15$
$8 + 1 + 6 = 15$

Add diagonally (from corner to corner):

$4 + 5 + 6 = 15$
$2 + 5 + 8 = 15$

Here is a larger one. Check that this is a magic square by adding across, then down and finally along the diagonals.

16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

Exercise 4a

1. Complete this magic square. The result of adding across, down and diagonally is 34.
Hint. Find the number in square *a* first, then *b, c, d, e, f, g* and *h* in that order.

1	*h*	7	12
g	4	*a*	*e*
10	5	16	*d*
8	*f*	*b*	*c*

2. Complete this square.

	14	12	7
4		9	
		8	
16			10

3. Try to design a magic square made up of 16 small squares.

Here is some more fun with figures.

Odd and Even Numbers

All the whole numbers are either odd or even:

1	2	3	4	5	6	7	8	9	10	and so on.
odd	even	odd	even	odd	even	odd	even	odd	even	and so on.

Note that even numbers can be divided by 2 and that odd numbers cannot be divided by 2.

Exercise 4b

Say whether the following numbers are odd or even:
22, 64, 37, 192, 380, 13, 125, 98.

The first ten odd numbers are:
1, 3, 5, 7, 9, 11, 13, 15, 17, 19.

$$
\begin{array}{lrcl}
1 & & = 1 = & 1^2 \\
1+3 & & = 4 = & 2^2 \\
1+3+5 & = 4+5 & = 9 = & 3^2 \\
1+3+5+7 & = 9+7 & = 16 = & 4^2 \\
1+3+5+7+9 & = 16+9 & = 25 = & 5^2 \\
1+3+5+7+9+11 & = 25+11 & = 36 = & 6^2 \\
1+3+5+7+9+11+13 & = 36+13 & = 49 = & 7^2 \\
1+3+5+7+9+11+13+15 & = 49+15 & = 64 = & 8^2 \\
1+3+5+7+9+11+13+15+17 & = 64+17 & = 81 = & 9^2 \\
1+3+5+7+9+11+13+15+17+19 & = 81+19 & = 100 = & 10^2
\end{array}
$$

Notice the neat way of writing these results in the right-hand column.

3^2 is read as '3 squared'.

From the table you can quite easily see that the sum of the first 3 odd numbers is 3^2, the sum of the first 7 odd numbers is 7^2, etc. In fact, you could write that the sum of the first x odd numbers is x^2.

Exercise 4c

1. What is the sum of the first 12 odd numbers?
Check your answer by addition.

2. A set of odd numbers adds up to 225. How many are there?

Magic squares and these sums of odd numbers are just two of the remarkable discoveries that can be made about numbers.

The Missing Number Game

Think of a number. Double it. Add 10. Halve the result. What is your answer? Take 5 from this answer and you will get the number you first thought of.
A little thought will tell you how this works.

Try this. Think of a number. Add 7. Multiply by 3. Take away twice the number you first thought of. Add 4. Take away the number you first thought of. Whatever the number was your answer will be 25. Find out how this works and make up some games like this.

Notation, or the Way in which Numbers are Written

Our method of writing numbers has taken thousands of years to reach its present state. Before it was introduced addition, subtraction, multiplication and division of numbers proved very difficult.
How do we write numbers?

	1000	100	10	1	
(a)	2	7	8	6	Two thousand, seven hundred and eighty-six.
(b)	7	6	2	8	Seven thousand, six hundred and twenty-eight.
(c)		3	0	9	Three hundred and nine.
(d)	4	0	0	7	Four thousand and seven.

Each separate figure is called a digit.

In **(a)** each digit has a value according to its place. In the right-hand column, the 6 means 6 units; the 8 means 8 tens or 80, the 7 means 7 hundreds or 700 and the 2 means 2 thousands or 2,000.

In **(b)** the same digits are used, but this time they are in different places and therefore have different values.

In **(c)** there are no tens and an '0' is introduced in the tens column.

In **(d)** there are no hundreds or tens.

Here is a really large number:

37,341,206. Thirty-seven million, three hundred and forty-one thousand, two hundred and six.

Notice that the commas divide the number up into groups of three from right to left and that the words 'million' and 'thousand' occur with the commas.

Exercise 4d

Write out in words the following numbers:

1. 37
2. 307
3. 4,265
4. 5,017
5. 36,004
6. 213,420
7. 200,008
8. 4,321,201
9. 34,070,302
10. 79,007,026

Write out in numbers:

1. Ninety-three.
2. Four hundred and thirty-six.
3. Seven hundred and two.
4. Three thousand two hundred and fifty-five.
5. Six thousand and four.
6. Seventy thousand two hundred and twenty.
7. One hundred and four thousand and nine.
8. Three million, four hundred and thirty-one thousand, eight hundred and fifty-nine.
9. Fourteen million.
10. Nine million, forty thousand and two.

(a) ADDITION

Example

24 + 131 + 19 + 3,062

Write these numbers down thus:

$$\begin{array}{r} 24\ + \\ 131 \\ 19 \\ \underline{3062} \\ 3236 \end{array}$$

Do not use carrying figures if you can do without them.

Exercise 4e

1. 3 + 19 + 147 + 18
2. 37 + 14 + 288 + 306
3. 7 + 29 + 3468 + 128
4. 14 + 239 + 450 + 126
5. 796 + 823 + 14 + 117
6. 1206 + 4077 + 9874
7. 3146 + 2987 + 4116 + 378
8. 1118 + 3004 + 2355 + 103
9. 34702 + 369 + 9037 + 5355
10. 11407 + 8326 + 4187 + 932.

(b) SUBTRACTION

Example

3092 — 476

Write the numbers down thus with the units under one another:

$$\begin{array}{r} 3092\ - \\ 476 \\ \hline 2616 \end{array}$$

Always check your answer by adding the bottom two lines, and the answer should be the top line. Remember that the larger number should be above the smaller number.

Exercise 4f

1. 3642 — 2131
2. 47 — 19
3. 106 — 88
4. 263 — 187
5. 4323 — 59
6. 7106 — 3214
7. 3009 — 2485
8. 7000 — 2682
9. 14321 — 8064
10. Take 329 from 932.
11. What is the difference between 37 and 142?
12. 7631 — 7594

Exercise 4g

PROBLEMS ON ADDITION AND SUBTRACTION

1. Before a journey the speedometer of a car read 21,862 miles. At the end it read 22,407 miles. What was the length of the journey?

2. Form six different numbers from the digits 2, 8 and 7 and then add them all together.

3. John has 277 foreign stamps and Bill has 346. How many have they altogether?

4. On a certain Saturday the attendances at three football grounds were: Chelsea, 52,748; West Ham, 38,021; Charlton, 13,347. How many people attended the three matches altogether? How many more people attended Chelsea than West Ham? How many people attended West Ham and Charlton? By how many did the attendance at Chelsea exceed the total attendance at the other two matches?

5. At the beginning of a quarter an electricity meter read 31,003 units; at the end of the quarter it read 48,342 units. How many units were consumed during the quarter?

6. Jack has 76 marbles. Bill has 37 and Tom has 36. Has Jack got more marbles than Bill and Tom put together? If so, how many more has he got?

7. A man was born in 1893. How old was he in 1959?

8. A gardener planted 386 bulbs. Later he counted 327 blooms. How many bulbs failed to flower? When he dug them up he found 349. How many did he lose?

9. In a school of 982 boys, 635 can play soccer and 483 can play rugger. 220 boys can play neither game. How many can play both?

10. A school serves 1,672 dinners in one week. From Monday to Thursday the number of dinners were 351, 326, 308, 297. How many were served on Friday?

(c) MULTIPLICATION

Tables

$$8 + 8 + 8 + 8 + 8 + 8 + 8 = 56$$
$$8 \times 7 = 56$$

Here are two ways of writing down the same thing. They show that multiplication is just a shorter way of adding up a set of numbers that are all the same. The answer 56 is not worked out every time, it is obtained from the multiplication tables which should be learned by heart.

A table is built up as follows:

$$\begin{array}{ll} 1 \times 7 = 7 & \\ 2 \times 7 = 7 + 7 = 14 & (1 \times 7 + 1 \times 7 = 2 \times 7) \\ 3 \times 7 = 14 + 7 = 21 & (2 \times 7 + 1 \times 7 = 3 \times 7) \\ 4 \times 7 = 21 + 7 = 28 & (3 \times 7 + 1 \times 7 = 4 \times 7) \end{array}$$

$5 \times 7 = 28 + 7 = 35$
$6 \times 7 = 35 + 7 = 42$
$7 \times 7 = 42 + 7 = 49$
$8 \times 7 = 49 + 7 = 56$
$9 \times 7 = 56 + 7 = 63$
$10 \times 7 = 63 + 7 = 70$
$11 \times 7 = 70 + 7 = 77$
$12 \times 7 = 77 + 7 = 84$

Build up all the tables from 1 to 12 and make a table square. This is useful while you are learning the tables. You can check your work from the table below.

A Table Square

						B						
	1	2	3	4	5	6	7	8	9	10	11	12
	2	4	6	8	10	12	14	16	18	20	22	24
	3	6	9	12	15	18	21	24	27	30	33	36
	4	8	12	16	20	24	28	32	36	40	44	48
	5	10	15	20	25	30	35	40	45	50	55	60
	6	12	18	24	30	36	42	48	54	60	66	72
A	7	14	21	28	35	42	49	56	63	70	77	84
	8	16	24	32	40	48	56	64	72	80	88	96
	9	18	27	36	45	54	63	72	81	90	99	108
	10	20	30	40	50	60	70	80	90	100	110	120
	11	22	33	44	55	66	77	88	99	110	121	132
	12	24	36	48	60	72	84	96	108	120	132	144

To find 7×6. Find the 7 at A in the left-hand column.
Find the 6 at B in the top row.
Where row A crosses column B you will find 42: $7 \times 6 = 42$.
Now look up 6×7 in the same way and you will see that

$$6 \times 7 = 42$$

Hence $6 \times 7 = 7 \times 6$.

Exercise 4h

Write down the answers to the following multiplication sums from memory. If you cannot remember the answers use your table square.

1. 3×4, 5×7, 6×3, 8×7, 6×0
2. 12×3, 7×8, 6×2, 9×9, 0×5
3. 5×5, 4×12, 3×11, 7×11
4. 2×7, 12×6, 4×8, 7×9
5. 2×3, 20×3, 3×20, 20×30
6. 5×4, 5×40, 40×5, 40×50
7. 7×10, 7×100, 7×1000
8. $2 \times 3 \times 4$, $20 \times 3 \times 4$, $20 \times 30 \times 4$, $20 \times 3 \times 40$, $2 \times 0 \times 3 \times 4$

Short Multiplication

$$\begin{array}{r} 24\ \times \\ \underline{7} \\ 168 \end{array}$$

As before, avoid carrying figures if you can manage without them.

Exercise 4j

1. 23×3
2. 29×7
3. 46×6
4. 278×8
5. 1479×2
6. 360×7
7. 103×4
8. 507×5
9. 1603×9
10. 142×12
11. 3006×11
12. 9183×9

Study this:

(a) $14 \times 21 = 14 \times 20 + 14 \times 1$
$= 280 + 14$
$= 294$

(b) $13 \times 99 = 13 \times 100 - 13 \times 1$
$= 1300 - 13$
$= 1287$

Exercise 4k

Try these examples by the above method.

1. 17×15
2. 13×24
3. 36×19
4. 16×99
5. 35×98
6. 78×102 *Hint*. $78 \times 100 + 78 \times 2$
7. 19×51
8. 129×1001
9. 136×72
10. 76×48 *Hint*. $76 \times 50 - 76 \times 2$

Long Multiplication

$$\begin{array}{rl} 763\times & \\ 24 & \\ \hline 15260 & (763 \times 20) \\ 3052 & (763 \times 4) \\ \hline 18312 & (763 \times 24) \end{array}$$

Always multiply by the highest digit first. In this case by 2 which is really 20. This is just an easier way of setting out examples like (a) in the section above.

Here is a harder example:

$$\begin{array}{r} 798 \times \\ 306 \\ \hline 239400 \\ 4788 \\ \hline 244188 \end{array}$$

Notice that there is no line corresponding to the 0 in 306.

Exercise 4l

1. 87×56
2. 367×48
3. 407×27
4. 623×125

5. 346×208
6. 802×509
7. 989×79
8. 345×54
9. $1{,}760 \times 47$
10. $1{,}092 \times 112$.

Exercise 4m

PROBLEMS ON MULTIPLICATION

1. How many days are there in 52 weeks?

2. A petrol drum holds 5 gallons. How many gallons are contained in 37 full drums?

3. An aircraft carries 47 passengers. If it makes 4 flights a week, how many passengers will it carry in 7 weeks if it is full for every flight?

4. A man travels 17 miles to work, and 17 miles back. How far does he travel in 6 working days?

5. An orchard contains 27 rows of apple trees, with 37 in each row. How many trees are there?

6. A car travels 34 miles on a gallon of petrol. How far will it go in using 27 gallons.

7. A train has seven carriages. Each carriage has 12 compartments each holding 8 people. How many passengers can be seated in the train?

8. A school has 37 classrooms each seating 33 pupils. How many chairs are required? If 5 new classrooms each seating 35 are built, what is now the total number of chairs required?

9. A book contains 219 pages. How many pages will there be in 27 of these books?

10. A factory employs 273 men. Each man does 9 hours' work a day, for 5 days a week. How many 'man-hours' of work could be done in a full year of 49 weeks 2 days if there was no absence?

(d) DIVISION

1. $7 \times 3 = 21$ 2. $\frac{21}{7} = 3$ 3. $\frac{21}{3} = 7$

These statements are different ways of writing the same thing.

Exercise 4n

Write each of the following in two different ways.

1. $4 \times 3 = 12$ **2.** $\frac{36}{4} = 9$ **3.** $\frac{27}{3} = 9$ **4.** $\frac{112}{14} = 8$

5. $12 \times 9 = 108$

A gardener had 72 seedlings to plant in 3 rows. How many seedlings in each row?

$$\text{No. of seedlings per row} = \frac{72}{3} = 24 \qquad 3\,\overline{)72}^{\,24}$$

This problem could have been set in this way:

If a gardener plants out 72 seedlings with 24 in each row, how many rows will he have?

$$\text{No. of rows} = \frac{72}{24} = 3$$

Short Division

Short division is always used when dividing by numbers up to 12.

e.g. $\frac{747}{9} = 83$ $\qquad 9\,\overline{)747}^{\,83}$

The answers are not always exact.

e.g. $\frac{1603}{12} = 133\frac{7}{12}$ $\qquad 12\,\overline{)1603}^{\,133\frac{7}{12}}$

Note that the remainder is written as a fraction. As a general rule always write the remainder in this way when the calculation is just a numerical one. Always think carefully before you decide how to write the remainder. The following problem illustrates this point.

There are 27 eggs to be packed in boxes, each holding 6 eggs. How many boxes will be required and how many eggs will be left over?

$$\text{No. of boxes} = \frac{27}{6} \qquad 6\,\overline{)27}^{\,4 \text{ rem. } 3}$$

$= 4$ and 3 eggs left over.

It is clear that the remainder here is 3 whole eggs. To break these into fractions is not only pointless but very messy!

Exercise 4p

In numbers **1–10** express the remainders, if any, as fractions.

1. $\frac{144}{6}$

2. $\frac{178}{7}$

3. $\frac{5280}{3}$

4. $\frac{432}{12}$

5. $\frac{918}{9}$

6. $\frac{1004}{5}$

7. $\frac{1364}{11}$

8. $\frac{7134}{7}$

9. $\frac{3118}{8}$

10. $\frac{1643}{10}$

11. A taxi holds 4 passengers. How many taxis would be required for 28 passengers?

12. A piece of wire 54 inches long is cut into equal pieces each 7 inches long. How many pieces will there be, and how much wire is left over?

13. How many sweets will each boy have if 143 sweets are shared amongst 11 boys?

14. By what number must I divide 91 to get the answer 7?

15. A farmer has 7 pig-sties in which he puts 50 pigs. If there are the same number of pigs in each sty find this number and say how many pigs will be left outside.

16. A grocer weighs out 110 pounds of sugar in three-pound bags. How many bags will he have and how much sugar is left over?

17. A number is divided by 9. The answer is 14 and the remainder is 7. What is the number?

18. A secretary took down a letter of 300 words in 7 minutes. How many words to the minute is this?

19. A car travelled 288 miles on 9 gallons of petrol. How many miles did it travel per gallon?

20. How many bags of potatoes each weighing 10 pounds can be made up from 5 hundredweights of potatoes?

Long Division

Long division is used when dividing by numbers bigger than 12. It is done in exactly the same way as short division except that the working is set out more fully.

e.g. $\frac{3546}{23} = 154\frac{4}{23}$

$$
\begin{array}{r l}
154 & \\
23\,\overline{)\,3546} & \text{(a)} \\
\underline{23} & \text{(b)} \\
124 & \text{(c)} \\
\underline{115} & \text{(d)} \\
96 & \text{(e)} \\
\underline{92} & \text{(f)} \\
4 & \text{(g)}
\end{array}
$$

Here are the steps:

(a) 23 into 3 won't go. 23 into 35 goes 1. Write the 1 immediately above the 5 and write $23 \times 1 = 23$ underneath the 35.

(b) Subtract 23 from 35, leaving 12 and carry down the 4 from the top line.

(c) and **(d)** 23 into 124 goes 5 (finding the 5 and similar steps is the hardest part of long division, but you will soon improve with practice). Write down the 5 above the 4 and underneath the 124 write $23 \times 5 = 115$, subtract 115 from 124 and carry down the 6.

e and **f** are similar to **(c)** and **(d)**.

g After **(f)** there are no more numbers to carry down and so the 4 is the remainder which is best written as $\frac{4}{23}$.

Here is a harder example:

$\frac{51697}{73} = 708\frac{13}{73}$

$$
\begin{array}{r l}
708 & \\
73\,\overline{)\,51697} & \\
\underline{511} & \\
597 & \text{(a)} \\
\underline{584} & \\
13 &
\end{array}
$$

Note that in line **(a)** after the 9 is carried down 73 will not go into 59. *Write an* 0 *over the* 9 and carry down the 7. Then carry on as before.

Here is a problem:

2000 flower bulbs are to be packed into bags, each containing 144 bulbs. How many bags are required, and how many bulbs are left over?

$$\text{No. of bags} = \frac{2000}{144}$$

$$= 13 \text{ bags and } 128 \text{ bulbs left over}$$

$$\begin{array}{r}
13 \\
144\,\overline{)\,2000} \\
\underline{144} \\
560 \\
\underline{432} \\
128
\end{array}$$

Exercise 4q

In exercises **1–10** write the remainders, if any, as fractions.

1. $\dfrac{296}{14}$ **2.** $\dfrac{3752}{18}$ **3.** $\dfrac{9620}{26}$ **4.** $\dfrac{3917}{38}$

5. $\dfrac{43440}{121}$ **6.** $\dfrac{9342}{57}$ **7.** $\dfrac{1062}{17}$ **8.** $\dfrac{3602}{35}$

9. $\dfrac{24367}{92}$ **10.** $\dfrac{218808}{27}$

11. A car uses 1 gallon of petrol for every 27 miles. How much petrol does a motorist use in a year if he travels a total of 10,000 miles?

12. The product of two numbers is 11,396. If one of them is 28 what is the other?

13. A dealer bought 45 washing machines for a total of £1,485. How much did he pay for each machine?

14. 37 posts are placed in a straight line. If the distance between the first and the last is 504 feet, what is the distance between the posts?

15. A lorry carries 52 sacks of potatoes. If the load weighs 5,824 pounds, find the weight of each sack.

16. Change 63,360 inches into yards (1 yd. = 36 in.).

17. A ship covers 391 miles in 17 hours. What is the average speed?

18. A farmer can plant 380 cabbages in each row in a certain field. How many complete rows will there be when he has planted out 10,000 cabbages?

5 Angles (I)

An angle is an amount of turn.
If you stand facing one direction and you turn to face another direction you will have turned through an 'angle'.
Angles are measured in 'degrees' (written for example, like this: $10° =$ ten degrees).

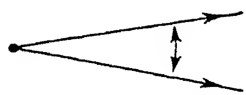


fig. 5.1

If you turn completely round once, and finish facing the original direction you will have turned through ONE REVOLUTION.

$$\text{ONE REVOLUTION} = 360° \text{ (360 degrees)}$$

Thus the minute hand of a clock turns through 360° in one hour.

Exercise 5a

1. How many degrees will a minute hand turn through in **(a)** $\frac{1}{4}$ hr. **(b)** $\frac{1}{2}$ hr. **(c)** $\frac{3}{4}$ hr. **(d)** 40 min. **(e)** 5 min. **(f)** 10 min. **(g)** 20 min. **(h)** 2 hr. **(i)** $1\frac{1}{2}$ hr. **(j)** 1 hr. 5 min.
2. How many degrees will an hour hand move through in **(a)** 1 hr. **(b)** 6 hr. **(c)** 12 hr. **(d)** 4 hr. **(e)** 30 min. **(f)** 1 min. **(g)** 10 min. **(h)** $1\frac{1}{2}$ hr. **(i)** 24 hr. **(j)** 11 hr.

Angle Arms

When an angle is turned through there must be a starting point for the turn and also a finishing point. Consider as an example the hour hand of a clock; if it starts when pointing to the 12 and turns until it points to the 3, then the hand has turned through 90°. The starting point in our example is a line from the centre of the clock pointing in the direction of the 12, and the finishing point is a line

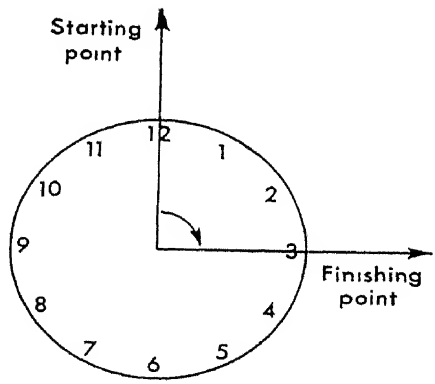


fig. 5.2

from the centre of the clock pointing in the direction of the 3, as in fig. 5.2. The lines drawn in these two directions are known as the 'arms' of the angle, and are necessary when we come to measure the size of the angle turned through.

Now it does not matter whether we are considering the hour hand of Big Ben or the hour hand of a tiny wrist-watch, the angle the hands turn through is the same in both cases. This leads to the very important fact that the size of the angle is not affected by the length of the angle arms.

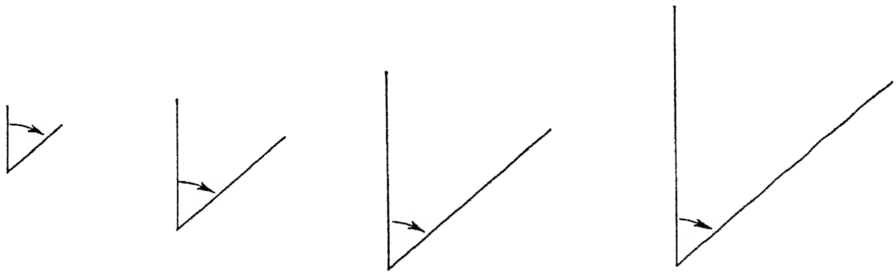


fig. 5.3

All these angles are the same size, but the lengths of the angle arms are different.

Exercise 5b

COMPASS BEARINGS

One important use made of angular measurement is in taking compass bearings. Assume you start facing North, how many degrees do you turn through if you turn *clockwise* to face (1) E. (2) W. (3) S. (4) NNE. (5) WSW. (6) SSW. (7) SE. (8) ENE. (9) WNW. (10) NNW.

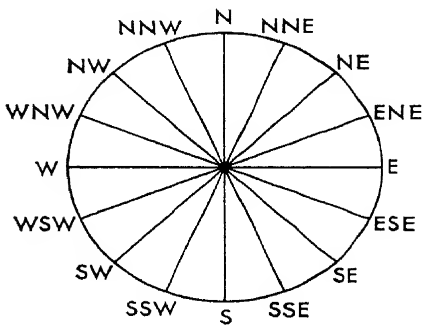


fig. 5.4

The Right Angle

The most important angle is 90°. This angle has a special name and is called a 'right angle' (right in geometry is short for upright). As you can see in fig. 5.5 it is the square corner that you met in chapter 2.

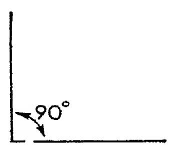


fig. 5.5

Drawers and windows for example will only work if their corners are 90°. To make sure that angles are exactly right angles engineers and carpenters use special tools to check them.

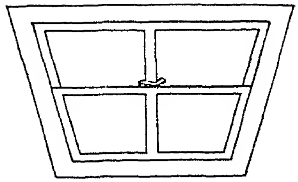


fig. 5.6

fig. 5.7 Engineer's ground cylindrical square

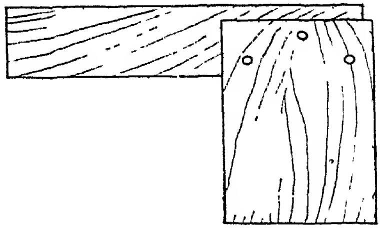


fig. 5.8 Carpenter's try square

You can make an accurate right angle for checking from a piece of paper as follows. Fold the paper once, this will give a perfect straight line. Call the ends of this line A and B. Fold the paper again so that A lies on B, and crease the paper again. You will then have a perfect right angle as shown in fig. 5.10.

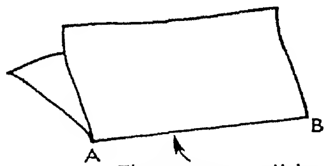


fig. 5.9

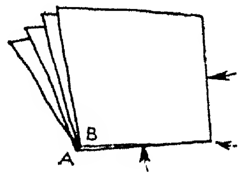


fig. 5.10

It is often necessary to name angles so that they can be picked out from other angles. There are three methods:

(1) By using the letter at the corner of the angle. The angle in fig. 5.11 would be angle B.

(2) By using the letters on the arms of the angle. The angle in fig. 5.11 would be angle ABC. This is the most usual method. Note that the middle letter of the three is the letter at the turning point.

(3) By using a letter inside the arms of the angle as in fig. 5.12.

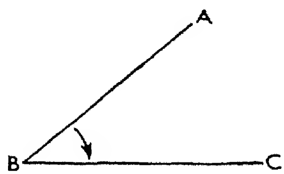


fig. 5.11

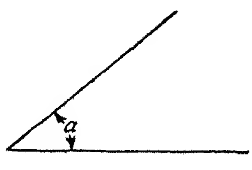


fig. 5.12

Exercise 5c

1. How many right angles are there in one revolution?

2. If two right angles are put together what do the arms which are not touching form?

3. Cut out a 3-inch square from squared paper. Name the four corners A, B, C, D in order. Fold along BD.
(a) What do you find about A and C?
(b) What can we say about the line BD?
(c) What can we say about angles ABD, DBC, BDA, BDC? Now fold along AC.
(d) What can we say about angles BAC, DAC, ACD, ACB? With your paper right angle check the angles where AC and BD cross.

The Protractor

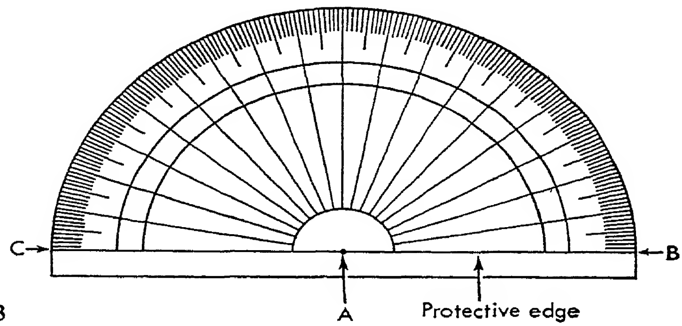


fig. 5.13

The instrument used to measure angles is called a protractor. A complete protractor is a full circle and is similar to fig. 5.16. The usual protractor used is only half of the full circle, fig. 5.13. In order to avoid damage an extra protective piece is added at the bottom. To use the protractor place it over the angle to be measured with the corner of the angle at the centre of the protractor (A)—stage 1. Swing the protractor round until one of the arms of the angle lies along either AB or AC, stage 2. Using this as the starting point count round in tens first, then the units of the degrees, until you reach the second arm, stage 3. This will give you the size of the angle.
An important note on using the protractor.
Along the measuring edge of the protractor there are two sets of numbers to help you. Do not get these numbers confused. One set is used when you start counting from C, and the other set is used when you start counting from B.

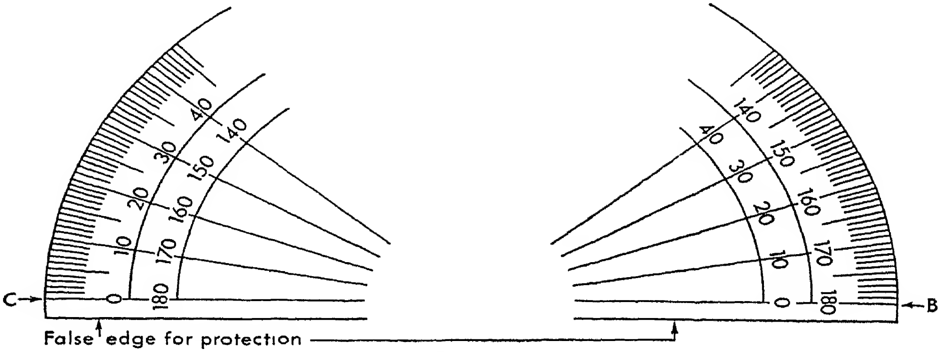
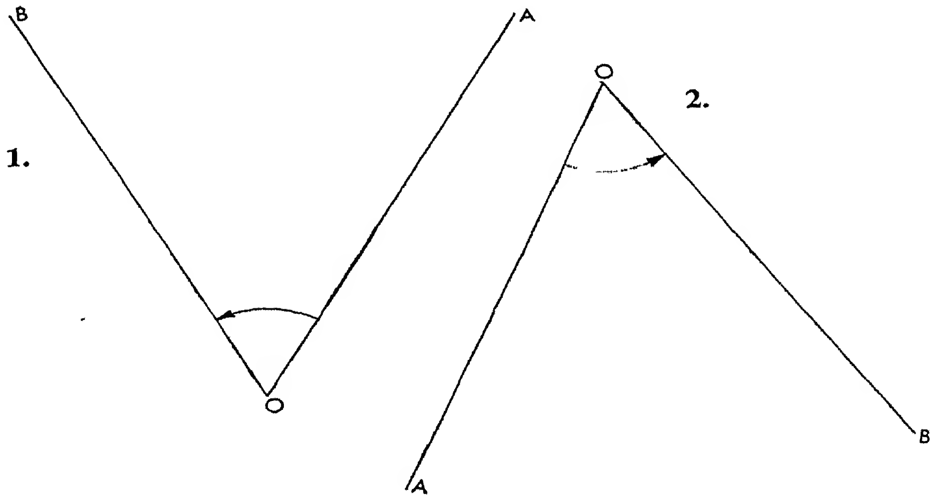


fig. 5.14

Fig. 5.14 shows the two ends of a protractor. If you start counting your angle round from C you will use the outer numbers, for you will count 10°, 20°, 30°, . . . etc. until you reach the second arm of the angle. If you have to start from B you will use the inner set of numbers, counting as before 10°, 20°, 30°, . . . etc.

Exercise 5d

Guess the size of the angles first, then measure them with a protractor. To help you guess the angle ask yourself if it is greater or smaller than a right angle. Is it greater or smaller than 2 right angles, etc.?



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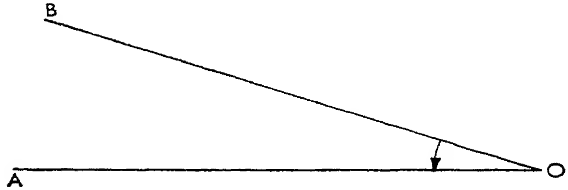

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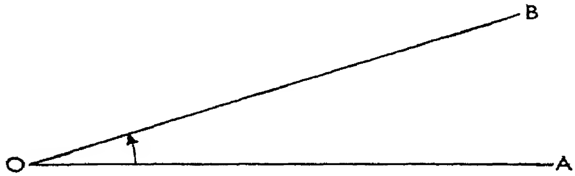

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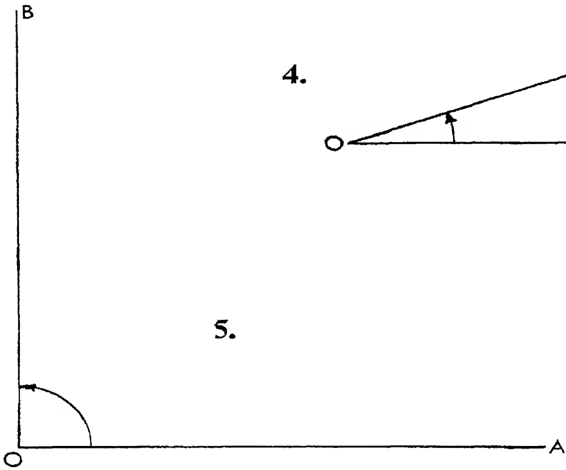

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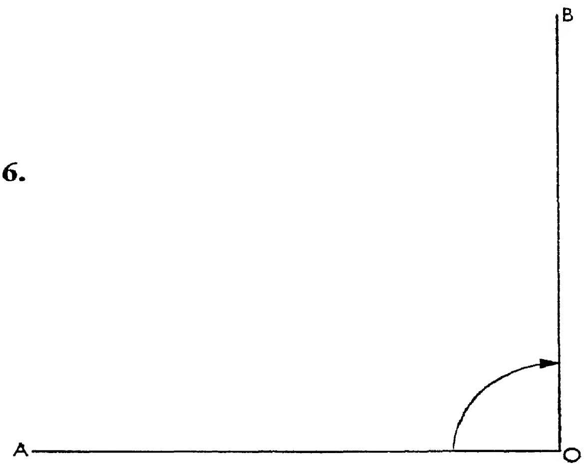

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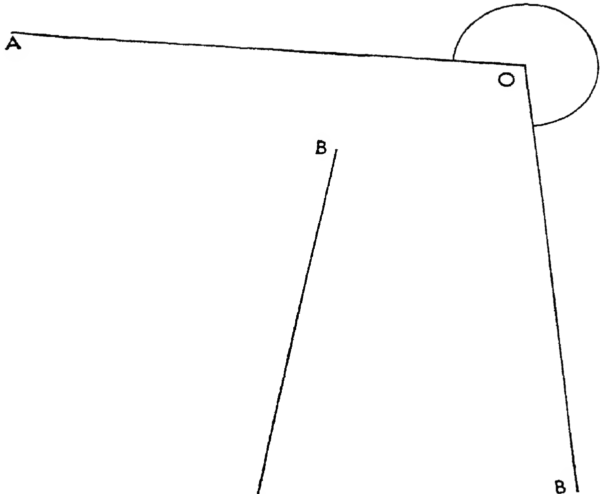

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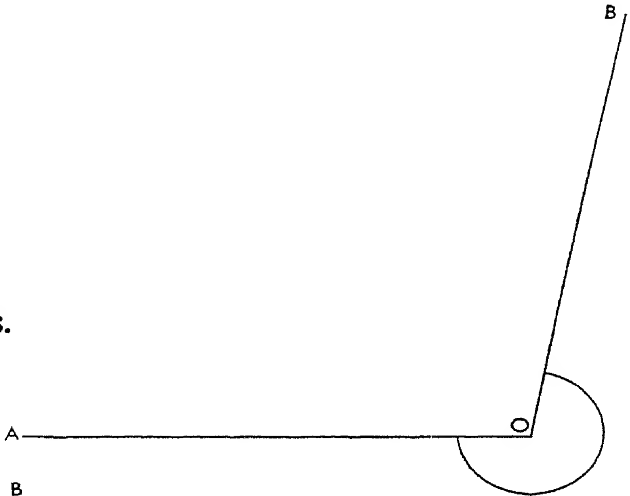
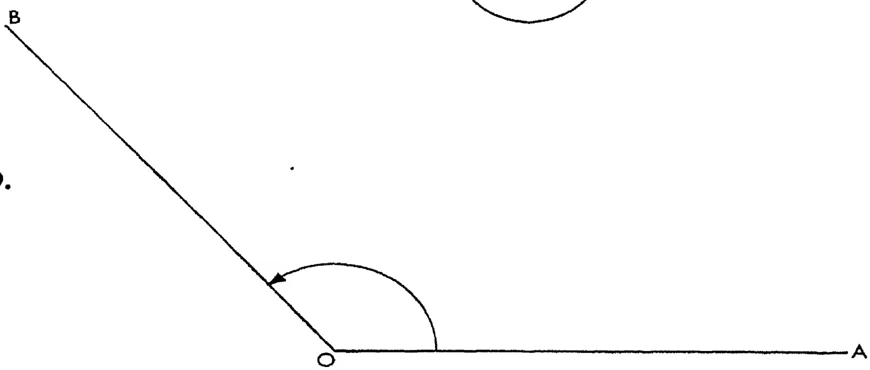


9.



10.

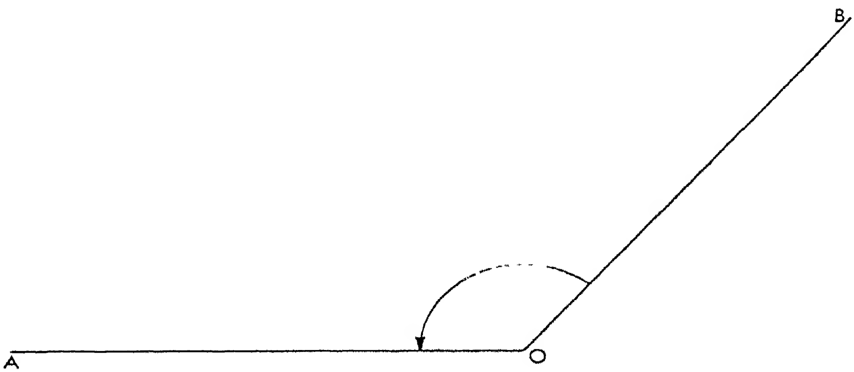


fig. 5.15

Exercise 5e

PLAYGROUND BEARINGS

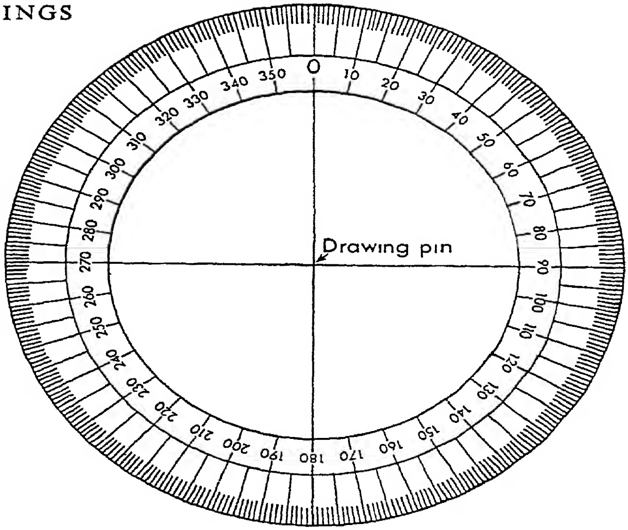


fig. 5.16

Ask your grocer for a cardboard box or a piece of thick cardboard. Draw out as large as possible the bearing card as illustrated in fig. 5.16 and stick it on the cardboard. Cut another strip of cardboard to make the pointer as shown and assemble to make a 'plane-table'. Your teacher will show you how to take bearings in the playground or from a high window.

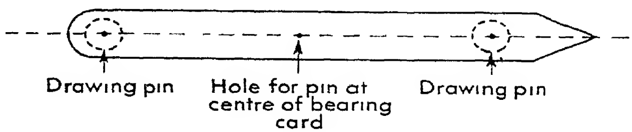


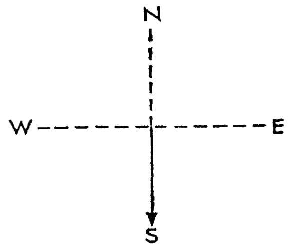
fig. 5.17

Bearings and Direction

There are several different methods of recording compass bearings. Here is a method where the north–south line is the starting point. The direction is given by 2 letters and a number, like this: S60°E. *To find the bearing*. (1) Face the direction of the first letter. (2) Turn through the angle indicated by the number towards the direction of the last letter.

e.g. (**a**) S60°E

(1) Face south. (2) Turn through 60° in an easterly direction.



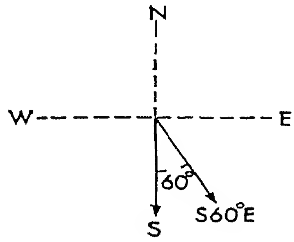
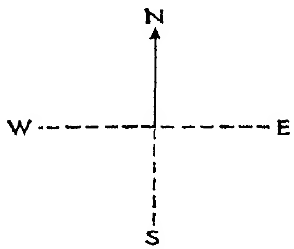


fig. 5.18

Example (**b**) N40°W

(1) Face north. (2) Turn through 40° towards the west.



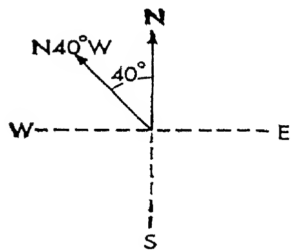


fig. 5.19

Exercise 5f

1. From London, Bristol is roughly 100 miles due west, and Grimsby is 130 miles due north. On a drawing, using 1 inch to 50 miles, find how far apart Grimsby and Bristol are.

2. A battleship is 4 miles due south of an island; a cruiser is due east of the island. The bearing of the cruiser from the battleship is N30°E. What is the range? Scale 1 in. to 1 mile.

3. From Norwich, Cromer is 24 miles due north and King's Lynn is 36 miles in a direction N77°W. How far is King's Lynn from Cromer? What is the bearing of King's Lynn from Cromer and what is the bearing of Cromer from King's Lynn? Scale 1 in. to 6 miles.

4. A ship proceeds 10 miles due north from a point A, then 12 miles N60°E, to a point B. How far is A from B? What direction is A from B? What direction is B from A? Scale 1 in. to 4 miles.

5. Ship X is 10 miles due north of ship Y. Ship X proceeds due west and ship Y proceeds in a direction N30°W. How far does Y travel before crossing the path of X? Scale 1 in. to 2 miles.

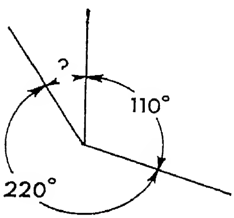
6 Angles (II)

You have already seen that if you turn through 360° you turn through one complete revolution and end up facing in the same direction as you started. If you make several small turns all in the same direction and finish up facing the original direction then the sum of the angles turned through will be 360°.
This is often expressed another way by saying 'the angle about a point is 360°'.

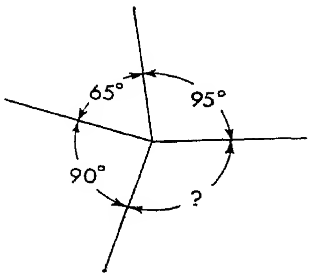
Exercise 6a

Find the missing angles in the following diagrams:

1.



2.



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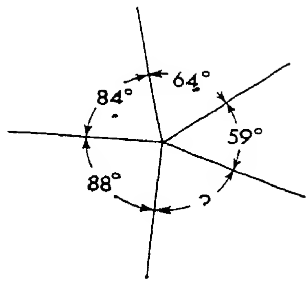

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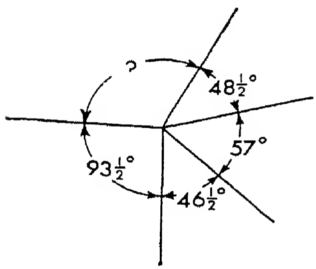

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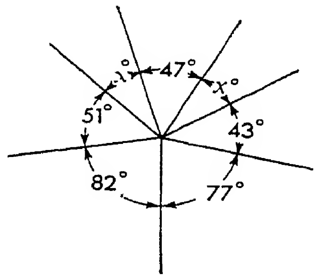


fig. 6.1

If you turn through half a revolution, you turn through 180°. If you were facing A then you turned through half a revolution to face B, then AOB would be in a straight line. This too is put in another way by saying 'the angle on a straight line is 180°'.

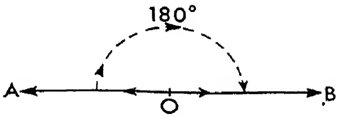


fig. 6.2

Exercise 6b

1. In which diagram is AOB a straight line?

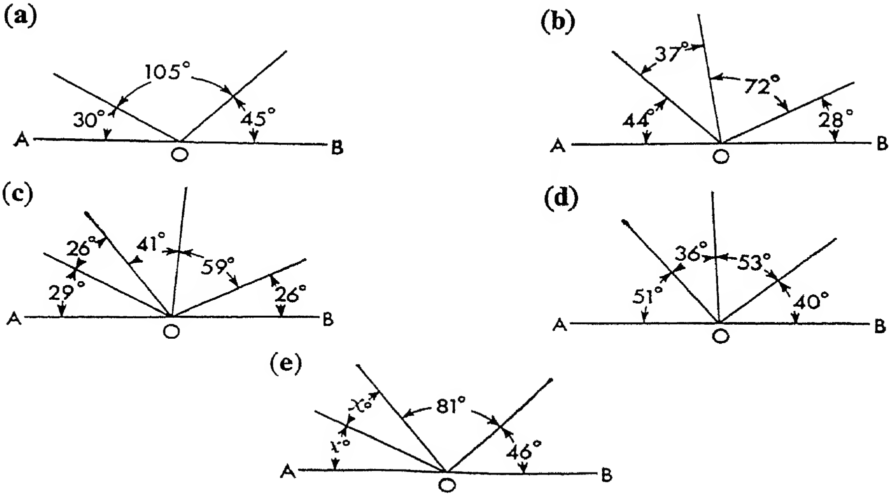


fig. 6.3

2. If AOB are all straight lines, find the missing angles.

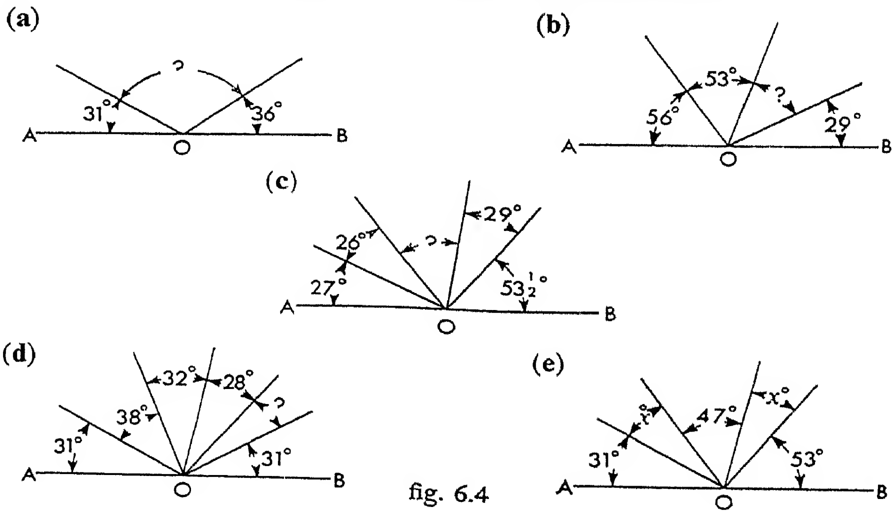


fig. 6.4

Some 'Angle Facts' of a Triangle

'Triangle' really means 3-angles. Look up the dictionary and find six other words beginning with 'TRI-' which have a connection with the number 3.

Cut out any large triangle from paper (coloured if possible) with sides about 7 or 8 inches. Place it with the longest edge at the bottom and mark the mid-points of the other two sides. Fold the triangle along the line joining these two points. The top 'apex' (or point) of the triangle should now touch the bottom side, at A′. Now fold carefully the remaining two corners of the triangle to touch A′.
You should now have a new shape.
What is it?

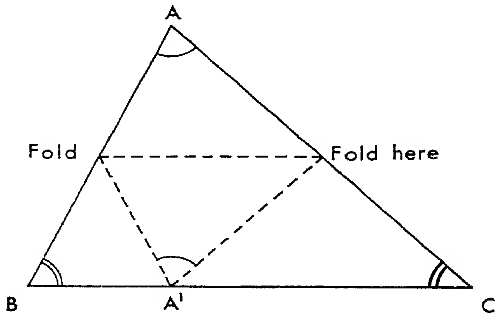


fig. 6.5

The point to notice at the moment is that the three angles of the triangle now fit along the base of the triangle. That is, they together form a straight line—i.e. they add up to 180°.
Stick the folded shape in your notebook and write a note underneath to show that this demonstrates that *the angles of a triangle add up to* 180°.

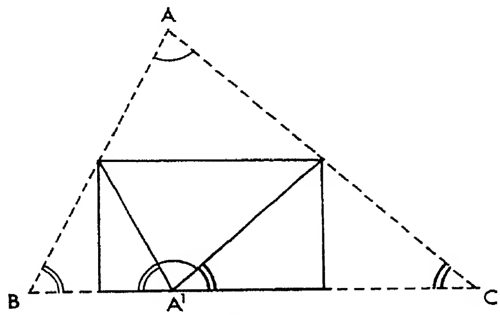
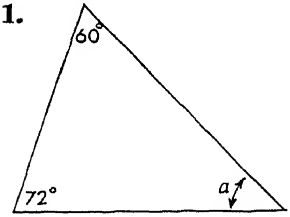


fig. 6.6

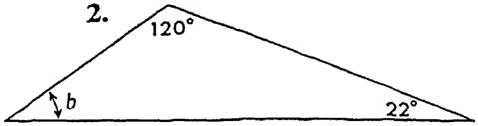
Exercise 6c

Find the missing angles in the following figures:

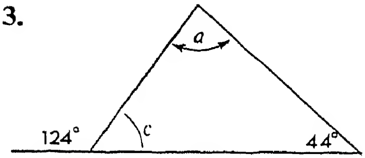
1.



2.



3.



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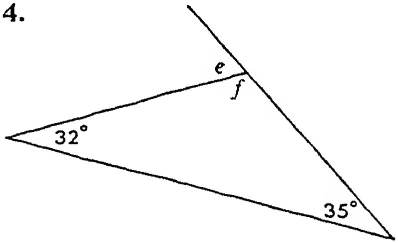

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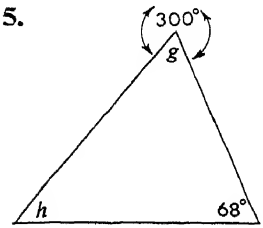

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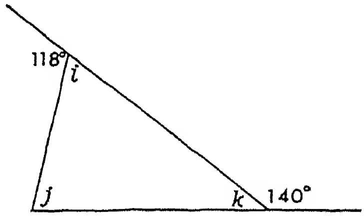

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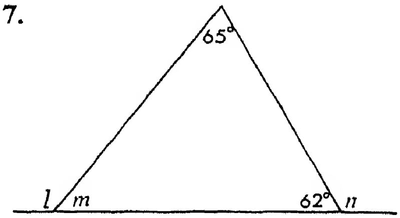

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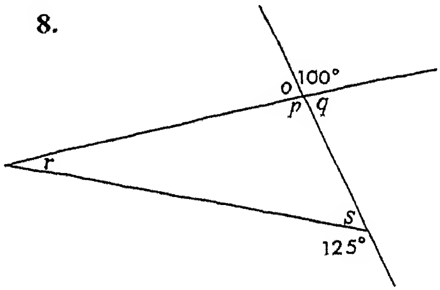
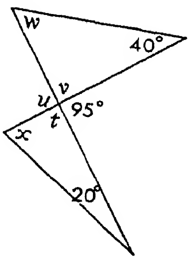


9.



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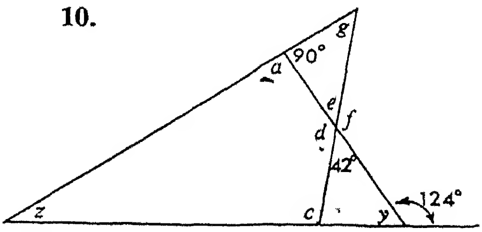


fig. 6.7

Angle Facts to be Learnt

(1) An angle less than 90° is called an *acute* angle.

(2) An angle more than 90° but less than 180° is called an *obtuse* angle.

(3) An angle more than 180° is called a *reflex* angle.

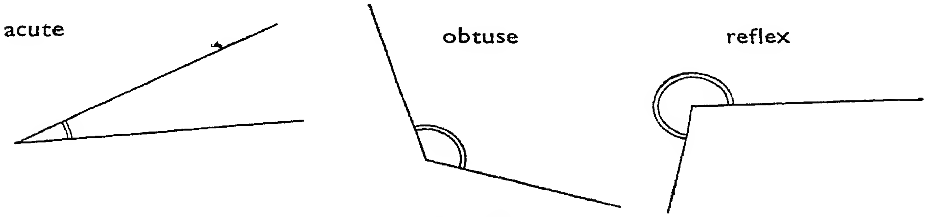


fig. 6.8

(4) Two angles which add up to 180° are called *supplementary* angles.
e.g. 30° and 150° are supplementary.
24° and 156° are supplementary.

(5) When two lines cross the opposite angles are equal and are called *vertically opposite* angles.
e.g. in fig. 6.9 a = c and a and c are vertically opposite. Also b = d, and b and d are vertically opposite.
To prove that vertically opposite angles are equal.
Since XY is a straight line a + b = 180°.
Since UV is a straight line a + d = 180°.
Thus a + b = a + d and therefore b = d.

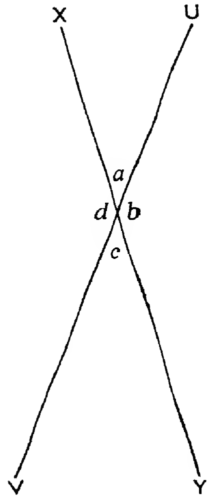


fig. 6 9

Exercise 6d

By exactly the same method as above prove a = c.

7 Fractions, Decimals & Percentages. Some everyday calculations

Vulgar Fractions

A B

If you measured the length of the line AB with a ruler marked only in inches, you would only be able to say that its length was between 3 and 4 inches, and that it was nearer to 4 than to 3 inches. In order to measure it more carefully, it is necessary to divide the inches into smaller divisions. These divisions are called fractions.

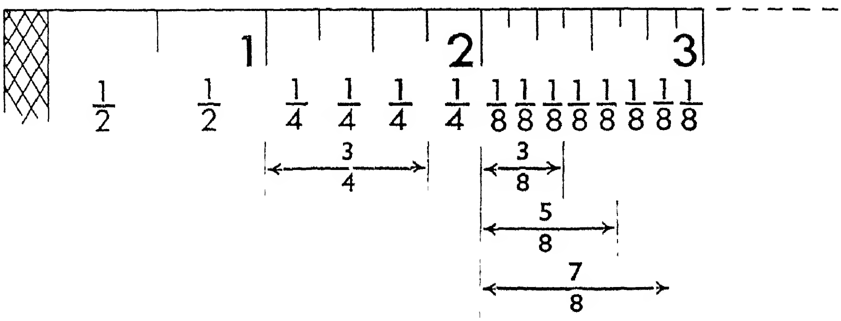


fig. 7.1

Look at the first inch of the ruler. It is divided into two equal parts or halves. (Written $\frac{1}{2}$.)
The second inch is divided into four equal parts or quarters. (Written $\frac{1}{4}$.) Notice that three of these parts is called $\frac{3}{4}$.
Now look carefully at the third inch. Into how many equal parts is it divided?

Here is a round cake. It is cut into four equal parts.
What is meant by '$\frac{3}{4}$ of the cake'?
$\frac{3}{4}$ of the cake is obtained by dividing the cake into four equal parts and taking three of them.
There is another meaning to $\frac{3}{4}$. It is the best way of writing 3 divided by 4.

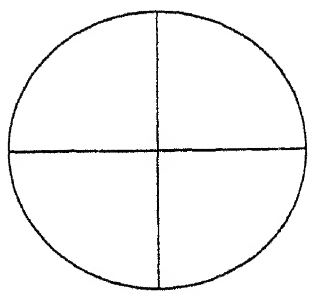


fig. 7.2

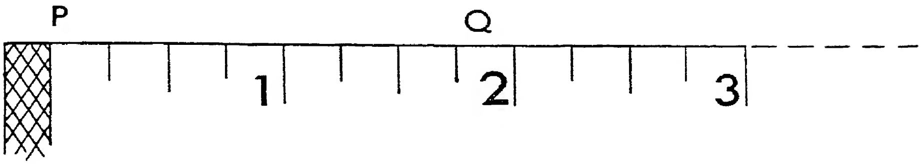


fig. 7.3

Here is a ruler marked out in quarter inches.
How many quarters are there in 3 inches?
Divide this number by 4.

In numbers this gives:

$$\begin{aligned} 3 \text{ divided by } 4 &= 12 \text{ quarters divided by } 4 \\ &= 3 \text{ quarters} \end{aligned}$$

or, $3 \div 4 = \frac{3}{4}$

Often whole numbers are combined with fractions. On the ruler above, the length PQ is one inch and three-quarters. This is written $1\frac{3}{4}$ and is called a mixed number because it contains both a whole number and a fraction.
Counting the number of quarters in PQ we get:

$$1\frac{3}{4} = \frac{7}{4}$$

i.e.

$$\begin{aligned} 1\frac{3}{4} &= \frac{4}{4} + \frac{3}{4} \\ &= \frac{7}{4} \end{aligned}$$

To do this more quickly multiply the whole number by 4 and add the result to 3.

e.g.

$$\begin{aligned} 3\tfrac{1}{2} &= \frac{(3 \times 2) + 1}{2} \\ &= \frac{7}{2} \end{aligned}$$

Such a fraction is called 'top-heavy' because the top number is bigger than the bottom.
When changing a 'top-heavy' fraction into a mixed number first divide the top by the bottom and write the remainder as a fraction.

e.g.

$$\frac{8}{3} = 2\frac{2}{3}$$

Exercise 7a

1. Measure the length of these lines using a ruler marked in eighths.

(a)
(b)
(c)
(d)
(e)

2. Measure the length of these lines using a ruler marked in tenths.

(a)
(b)
(c)
(d)
(e)

3. Write out in words the meaning of '$\frac{2}{3}$ of 1*s.*' How many pence is this?

4. Write out in words the meaning of £1$\frac{3}{8}$, and then give the answer in £ *s. d.*

5. Find in feet and inches:

(a) $1\frac{1}{4}$ ft. (b) $2\frac{1}{2}$ yd. (c) $\frac{5}{6}$ ft.
(d) $\frac{2}{3}$ yd. (e) $1\frac{3}{4}$ yd. (f) $3\frac{5}{12}$ ft.
(g) $3\frac{5}{6}$ yd.

6. Find in pounds and ounces:

(a) $1\frac{1}{2}$ lb. (b) $\frac{3}{4}$ lb. (c) $3\frac{1}{8}$ lb.
(d) $2\frac{5}{16}$ lb. (e) $\frac{7}{8}$ lb. (f) $7\frac{11}{16}$ lb.

7. Change into 'top-heavy' fractions:

(a) $2\frac{1}{4}$ (b) $1\frac{3}{7}$ (c) $3\frac{5}{8}$ (d) $1\frac{3}{16}$ (e) $2\frac{5}{9}$

(f) $7\frac{3}{10}$ (g) $12\frac{1}{3}$ (h) $3\frac{5}{16}$ (i) $4\frac{31}{100}$ (j) $2\frac{5}{18}$

8. Change into mixed numbers:

(a) $\frac{7}{4}$ (b) $\frac{12}{5}$ (c) $\frac{22}{7}$ (d) $\frac{19}{8}$ (e) $\frac{33}{2}$

(f) $\frac{16}{9}$ (g) $\frac{38}{21}$ (h) $\frac{123}{20}$ (i) $\frac{171}{10}$ (j) $\frac{433}{100}$

The Golden Rule of Fractions

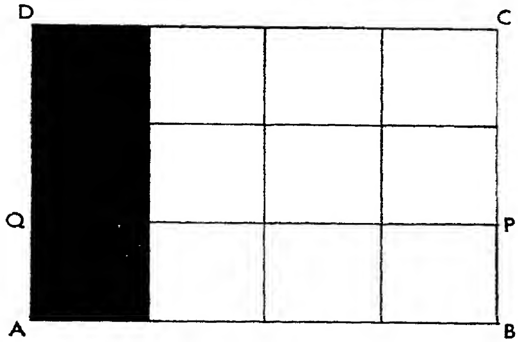


fig. 7.4

The rectangle ABCD is divided into 12 equal parts.
The shaded part is $\frac{3}{12}$ of the whole.
It is also $\frac{1}{4}$ of the whole.

$$\therefore \frac{1}{4} = \frac{3}{12} \qquad \text{(A)}$$

The bottom strip ABPQ is $\frac{4}{12}$ of the whole.
It is also $\frac{1}{3}$ of the whole

$$\therefore \frac{1}{3} = \frac{4}{12} \qquad \text{(B)}$$

In the same way the rectangle QPCD is $\frac{8}{12}$ of the whole.
It is also $\frac{2}{3}$ of the whole.

$$\therefore \frac{2}{3} = \frac{8}{12} \qquad \text{(C)}$$

Look carefully at line (A) again.
Notice that you could obtain the result by writing

$$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$$

By what numbers are the top and bottom multiplied to give the results in (B) and (C)?
This process can be extended.

e.g. $$\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$$

or, $$\frac{1}{4} = \frac{1 \times 7}{4 \times 7} = \frac{7}{28}$$

Exercise 7b

Fill in the missing numbers:

1. $\frac{1}{2} = \frac{}{4}$ **2.** $\frac{2}{3} = \frac{}{6}$ **3.** $\frac{5}{8} = \frac{10}{}$

4. $\frac{2}{3} = \frac{}{9}$ **5.** $\frac{7}{8} = \frac{21}{}$ **6.** $\frac{3}{7} = \frac{}{49}$

7. $\frac{5}{11} = \frac{}{66}$ **8.** $\frac{12}{13} = \frac{24}{}$ **9.** $\frac{7}{10} = \frac{}{1000}$

10. $\frac{13}{16} = \frac{52}{}$

Look at line (C) again, but this time write it the other way round:

$$\frac{8}{12} = \frac{2}{3}$$

This time divide both the top and bottom numbers on the left-hand side by 4 to get the right-hand side.
Usually we write:

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}} = \frac{2}{3}$$

This process is called 'cancelling' and it is done by dividing the top and the bottom of the fraction by the same number. When no more cancelling can be done the fraction is said to be in its lowest terms.

e.g. Express $\frac{45}{75}$ in its lowest terms.

$$\frac{\overset{\overset{3}{\cancel{9}}}{\cancel{45}}}{\underset{\underset{5}{\cancel{15}}}{\cancel{75}}} = \frac{3}{5}$$

Here the process is repeated. Division of top and bottom by 5 gives $\frac{9}{15}$ which is then cancelled by 3. When cancelling it is important to leave room above and below the fraction. Remember that untidy cancelling often leads to mistakes.

Exercise 7c

Fill in the missing numbers:

1. $\frac{4}{8} = \frac{}{2}$

2. $\frac{10}{12} = \frac{5}{}$

3. $\frac{10}{25} = \frac{2}{}$

4. $\frac{14}{21} = \frac{}{3}$

5. $\frac{18}{42} = \frac{3}{}$

6. $\frac{48}{108} = \frac{}{9}$

7. $\frac{24}{56} = \frac{3}{}$

8. $\frac{19}{57} = \frac{1}{}$

9. $\frac{180}{270} = \frac{}{3}$

10. $\frac{50}{125} = \frac{2}{}$

Reduce to lowest terms:

11. $\frac{9}{12}$

12. $\frac{12}{16}$

13. $\frac{36}{60}$

14. $2\frac{8}{10}$

15. $\frac{125}{150}$

16. $3\frac{15}{18}$

17. $\frac{21}{70}$

18. $\frac{90}{216}$

19. $7\frac{44}{121}$

20. $3\frac{75}{100}$

Copy this into your book:

The Golden Rule of Fractions states that when both the top and bottom of a fraction are multiplied (or divided) by the same number the value of the fraction remains unchanged.

The Four Rules for Fractions

ADDITION

e.g. $$\frac{1}{6} + \frac{3}{8}$$

Find the lowest number into which both 6 and 8 will divide. This is best done by taking the largest number 8 and going through the 8 times table until you get a number divisible by 6. The required number here is 24.

Now use the Golden Rule to change $\frac{1}{6}$ and $\frac{3}{8}$ into 24ths.

$$\frac{1}{6} = \frac{4}{24} \quad \text{and} \quad \frac{3}{8} = \frac{9}{24}$$

$$\therefore \frac{1}{6} + \frac{3}{8} = \frac{4}{24} + \frac{9}{24}$$

We can now add the 4 and the 9 together

$$\therefore \frac{1}{6} + \frac{3}{8} = \frac{13}{24}$$

To simplify the setting out we write:

$$\begin{aligned} \frac{1}{6} + \frac{3}{8} &= \frac{4 + 9}{24} \\ &= \frac{13}{24} \end{aligned}$$

Here are two more examples.

$$\begin{aligned} \frac{1}{2} + \frac{7}{8} &= \frac{4 + 7}{8} \\ &= \frac{11}{8} \\ &= 1\frac{3}{8} \end{aligned}$$

In this example the answer first appears as a top-heavy fraction which is then changed into a mixed number.

$$1\frac{3}{4} + 2\frac{7}{12} = 3\frac{9 + 7}{12}$$ Add the whole numbers first.

$$= 3\frac{16}{12}$$ Change top-heavy fraction to mixed number and add whole numbers. Reduce fraction part to lowest terms.

$$= 4\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{12}}}$$

$$= 4\frac{1}{3}$$

Exercise 7d

1. $\frac{1}{2} + \frac{1}{4}$, $\frac{3}{8} + \frac{1}{4}$, $\frac{5}{16} + \frac{3}{8}$, $\frac{1}{2} + \frac{5}{12}$, $\frac{3}{4} + \frac{3}{16}$
2. $\frac{2}{3} + \frac{1}{4}$, $\frac{1}{3} + \frac{2}{5}$, $\frac{1}{2} + \frac{2}{7}$, $\frac{2}{5} + \frac{1}{4}$, $\frac{1}{6} + \frac{3}{5}$
3. $\frac{3}{4} + \frac{7}{8}$, $\frac{9}{16} + \frac{5}{8}$, $\frac{2}{3} + \frac{5}{6}$, $\frac{7}{8} + \frac{1}{3}$, $\frac{1}{2} + \frac{3}{7} + \frac{5}{14}$
4. $2\frac{1}{4} + 1\frac{5}{16}$, $1\frac{2}{3} + 2\frac{1}{9}$, $3\frac{5}{6} + 1\frac{1}{3}$, $1\frac{2}{5} + 3\frac{3}{10}$, $4\frac{1}{4} + 3\frac{7}{16}$
5. $\frac{5}{6} + \frac{2}{9}$, $\frac{3}{8} + \frac{5}{6}$, $\frac{2}{3} + \frac{3}{5} + \frac{1}{6}$, $\frac{1}{8} + \frac{5}{6} + \frac{7}{12}$, $\frac{5}{12} + \frac{3}{8}$
6. $2\frac{3}{4} + 1\frac{1}{6}$, $3\frac{3}{5} + 1\frac{7}{10}$, $1\frac{1}{6} + 3\frac{5}{8} + 2\frac{1}{4}$, $3 + 1\frac{5}{16} + 2\frac{7}{24}$, $1\frac{11}{12} + 3 + 2\frac{3}{8}$
7. $3\frac{1}{3} + 2\frac{3}{5} + 1\frac{5}{6}$, $2\frac{7}{20} + \frac{11}{15} + 1\frac{13}{30}$, $1\frac{3}{7} + 2\frac{1}{5} + 1\frac{9}{10}$, $2\frac{5}{9} + 1\frac{4}{15} + \frac{2}{3}$, $1\frac{3}{4} + 2\frac{5}{12} + 3\frac{7}{16}$

SUBTRACTION

This is set out in the same way as addition.

e.g.

$$\frac{9}{16} - \frac{1}{4} = \frac{9-4}{16}$$
$$= \frac{5}{16}$$

Also

$$4\frac{3}{5} - 2\frac{1}{3} = 2\frac{9-5}{15}$$
$$= 2\frac{4}{15}$$

Subtract whole numbers first.

Note the following example very carefully.

$$5\frac{1}{6} - 1\frac{3}{4} = 4\frac{2-9}{12}$$
$$= 3\frac{14-9}{12}$$
$$= 3\frac{5}{12}$$

Deal with whole numbers first. Notice that 9 cannot be taken from 2. Take 1 from the 4 leaving 3. Change the 1 into twelfths, i.e. $1 = \frac{12}{12}$, and add those to the 2 twelfths, making 14. Now the 9 can be taken from 14 giving the answer.

Exercise 7e

1. $\frac{7}{8} - \frac{1}{2}$, $\frac{3}{4} - \frac{5}{16}$, $\frac{2}{3} - \frac{2}{9}$, $\frac{5}{6} - \frac{3}{8}$, $\frac{11}{12} - \frac{5}{18}$
2. $2\frac{9}{16} - 1\frac{1}{4}$, $3\frac{5}{14} - 1\frac{2}{7}$, $6\frac{5}{6} - 2\frac{1}{4}$, $3\frac{7}{9} - 1\frac{3}{4}$, $5\frac{14}{15} - 3\frac{7}{10}$
3. $3\frac{5}{8} - 1\frac{3}{4}$, $2\frac{1}{2} - 1\frac{5}{9}$, $3\frac{7}{16} - 2\frac{3}{8}$, $5\frac{1}{21} - 3\frac{2}{7}$, $6 - 2\frac{5}{16}$
4. $10 - 3\frac{5}{21}$, $4\frac{2}{15} - 3\frac{7}{10}$, $7\frac{12}{13} - 2\frac{5}{52}$, $6\frac{7}{18} - 4\frac{5}{12}$, $2\frac{7}{8} - 1\frac{5}{36}$

MULTIPLICATION

(a) By a whole number.

e.g.

$$\frac{3}{4}\times 6 = \frac{\overset{9}{\cancel{18}}}{\underset{2}{\cancel{4}}} = \frac{9}{2} = 4\frac{1}{2} \qquad \text{or} \quad \frac{3}{\underset{2}{\cancel{4}}}\times \overset{3}{\cancel{6}} = \frac{9}{2} = 4\frac{1}{2}$$

The second method is quicker because cancelling at the beginning makes the numbers smaller. This is another application of the 'Golden Rule'.

(b) By a fraction.

$\frac{2}{3}$ multiplied by $\frac{3}{4}$ means $\frac{2}{3}$ of $\frac{3}{4}$ and is written

$$\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} = \frac{1}{2}$$

$\frac{3}{7}\times\frac{2}{5} = \frac{6}{35}$ Nothing cancels here and so the two top numbers and the two bottom numbers are each multiplied together.

When multiplying by a mixed number it must first be changed into a top-heavy fraction.

e.g.

$$\frac{3}{4}\times 2\frac{2}{9} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}}\times\frac{\overset{5}{\cancel{20}}}{\underset{3}{\cancel{9}}}$$
$$= \frac{5}{3}$$
$$= 1\frac{2}{3}$$

Further

$$1\frac{3}{4}\times 1\frac{5}{7}\times 2\frac{5}{6} = \frac{\overset{1}{\cancel{7}}}{\underset{2}{\cancel{4}}}\times\frac{\overset{\overset{1}{\cancel{2}}}{\cancel{12}}}{\underset{1}{\cancel{7}}}\times\frac{17}{\underset{1}{\cancel{6}}}$$
$$= \frac{17}{2}$$
$$= 8\frac{1}{2}$$

Exercise 7f

1. $\frac{3}{5} \times 10$, $\frac{7}{8} \times 4$, $\frac{5}{9} \times 3$, $\frac{4}{7} \times 21$, $6 \times \frac{5}{48}$
2. $\frac{2}{3} \times \frac{5}{6}$, $\frac{3}{4} \times \frac{8}{9}$, $\frac{7}{16} \times \frac{4}{5}$, $\frac{13}{21} \times \frac{7}{26}$, $\frac{3}{10} \times \frac{25}{27}$
3. $2\frac{1}{2} \times \frac{2}{5}$, $1\frac{3}{4} \times \frac{2}{7}$, $6 \times 3\frac{1}{3}$, $3\frac{3}{4} \times 1\frac{1}{5}$, $\frac{3}{10} \times 6 \times 2\frac{2}{9}$
4. $7\frac{1}{2} \times 1\frac{2}{5}$, $3\frac{1}{5} \times 3\frac{1}{8}$, $4\frac{1}{8} \times 1\frac{3}{11}$, $6\frac{3}{4} \times 2\frac{5}{9}$, $3\frac{1}{7} \times \frac{14}{33}$
5. $\frac{3}{4} \times \frac{9}{11} \times 3\frac{2}{3}$, $\frac{7}{8} \times 5 \times 1\frac{3}{25}$, $12\frac{1}{2} \times \frac{2}{17} \times \frac{34}{175}$, $\frac{6}{13} \times 3\frac{5}{7} \times 2\frac{4}{5}$

DIVISION

What does $6 \div \frac{3}{4}$ mean?

$12 \div 3$ could mean 'How many 3's in 12?' and could be written $\frac{12}{3}$.

Write $6 \div \dfrac{3}{4} = \dfrac{6}{\frac{3}{4}}$ i.e., how many $\frac{3}{4}$'s in 6?

$$= \dfrac{6 \times \dfrac{4}{3}}{\dfrac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \dfrac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}}$$

Using the Golden Rule and multiplying top and bottom by the same number, in this case $\frac{4}{3}$.

$$= \overset{2}{\cancel{6}} \times \frac{4}{\underset{1}{\cancel{3}}} \qquad (1)$$

Now look at: $\frac{5}{8} \div 10 = \dfrac{\frac{5}{8}}{10}$

$$= \dfrac{\dfrac{5}{8} \times \dfrac{1}{10}}{\overset{1}{\cancel{10}} \times \dfrac{1}{\underset{1}{\cancel{10}}}}$$

$$= \frac{\overset{1}{\cancel{5}}}{8} \times \frac{1}{\underset{2}{\cancel{10}}} \qquad (2)$$

$$= \frac{1}{16}$$

Finally $$3\frac{3}{4} \div 6\frac{1}{4} = \frac{\frac{15}{4}}{\frac{24}{4}}$$

$$= \frac{\frac{15}{4} \times \frac{4}{25}}{\frac{\overset{1}{\cancel{25}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{25}}}}$$

$$= \frac{\overset{3}{\cancel{15}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{5}{\cancel{25}}} \qquad (3)$$

$$= \frac{3}{5}$$

Examine lines (1), (2) and (3) and you will see that they obey the general rule for dividing fractions which is:

When dividing one fraction by another turn the second fraction upside down and change the division sign to a multiplication sign.

Here are two examples to illustrate the rule which gives a more direct method than the one used above.

$$2\frac{2}{5} \div 6 = \frac{12}{5} \div \frac{6}{1}$$ Divide the 6 by 1 to make a fraction of it.

$$= \frac{\overset{2}{\cancel{12}}}{5} \times \frac{1}{\underset{1}{\cancel{6}}}$$ Turn $\frac{6}{1}$ upside down and change $\div$ into $\times$.

$$= \frac{2}{5}$$

Also
$$2\frac{4}{5} \div 1\frac{13}{15} = \frac{14}{5} \div \frac{28}{15}$$
$$= \frac{\overset{1}{\cancel{14}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{28}}}$$
$$= \frac{3}{2}$$
$$= 1\frac{1}{2}$$

Exercise 7g

1. $\frac{2}{3} \div \frac{5}{6}$, $\frac{7}{8} \div \frac{7}{10}$, $\frac{4}{5} \div \frac{16}{25}$, $\frac{2}{3} \div \frac{3}{5}$, $\frac{3}{8} \div \frac{5}{16}$
2. $6 \div \frac{3}{4}$, $10 \div \frac{5}{8}$, $14 \div \frac{7}{15}$, $12 \div 1\frac{1}{2}$, $16 \div 1\frac{3}{5}$
3. $\frac{9}{16} \div 3$, $\frac{14}{15} \div 21$, $\frac{8}{9} \div 24$, $\frac{9}{10} \div 21$, $\frac{25}{51} \div 15$
4. $\frac{7}{8} \div \frac{3}{4}$, $\frac{5}{6} \div \frac{15}{16}$, $\frac{7}{10} \div \frac{5}{7}$, $\frac{3}{16} \div \frac{3}{80}$, $\frac{9}{14} \div \frac{2}{7}$
5. $1\frac{3}{4} \div \frac{7}{8}$, $\frac{5}{16} \div 2\frac{1}{2}$, $3\frac{1}{4} \div 1\frac{5}{8}$, $4\frac{3}{4} \div 1\frac{1}{17}$, $2\frac{1}{3} - \frac{5}{9}$
6. $1\frac{7}{8} \div 1\frac{3}{5}$, $5\frac{1}{4} \div 1\frac{2}{7}$, $9\frac{1}{3} \div 1\frac{2}{5}$, $2\frac{3}{8} \div 38$, $14 \div 2\frac{1}{3}$

FRACTIONS OF GIVEN QUANTITIES

e.g. Express $\frac{2}{3}$ of 2*s.* in pence.

$$\frac{2}{3} \text{ of } 2s. = \frac{2}{\underset{1}{\cancel{3}}} \times 2 \times \overset{4}{\cancel{12}}d.$$

Change 'of' to '$\times$' and reduce the shillings to pence.

$$= 16d.$$

Express $\frac{7}{8}$ of $2\frac{1}{2}$ lb. in lb. and oz.

$$\frac{7}{8} \text{ of } 2\frac{1}{2} \text{ lb.} = \frac{7}{8} \times 2\frac{1}{2} \times 16 \text{ oz.}$$

$$= \frac{7}{\underset{1}{\cancel{8}}} \times \frac{5}{\underset{1}{\cancel{2}}} \times \overset{\overset{1}{\cancel{2}}}{\cancel{16}}$$
$$= 35 \text{ oz.}$$
$$= 2 \text{ lb. } 3 \text{ oz.}$$

Exercise 7h

Express the following in the units given.

1. $\frac{3}{4}$ of 1*s.* in pence.
2. $\frac{7}{8}$ of £1 in shillings and pence.
3. $\frac{9}{16}$ of 2 lb. in lb. and oz.
4. $\frac{4}{7}$ of 1 cwt. in lb.
5. $\frac{3}{4}$ of 1 yd. in ft. and in.
6. $\frac{3}{16}$ of £1 in *s. d.*
7. $\frac{2}{3}$ of £1 in *s. d.*
8. $\frac{3}{4}$ of 1 m. in yd.
9. $\frac{5}{7}$ of 3 st. in lb.
10. $\frac{3}{8}$ of 2 yd. in ft. in.
11. $\frac{7}{20}$ of 1 hr. in min.
12. $\frac{3}{14}$ of 1 week in days.
13. $\frac{3}{5}$ of 12*s.* 6*d.* in *s. d.*
14. $\frac{7}{8}$ of £1 12*s.* in *s.*
15. $\frac{7}{16}$ of £5 in *s. d.*
16. $\frac{5}{11}$ of 2 m. in yd.
17. $\frac{3}{5}$ of 8 st. 3 lb. in st. lb.
18. $\frac{5}{7}$ of $2\frac{1}{2}$ ton in cwt. and lb.
19. $\frac{7}{3}$ of $2\frac{1}{2}$ yd. in ft. in.
20. $\frac{19}{16}$ of $3\frac{1}{4}$ lb. in lb. oz.

EXPRESSING ONE QUANTITY AS A FRACTION OF ANOTHER

e.g. Express 2*s.* 6*d.* as a fraction of £1 2*s.* 6*d.*

$$\text{Required fraction} = \frac{2s.\ 6d.}{£1\ 2s.\ 6d.}$$

$$= \frac{\overset{1}{\cancel{5}}\ \text{sixpences}}{\underset{9}{\cancel{45}}\ \text{sixpences}} \quad \left\{ \begin{array}{l} \text{Change both top and bot-} \\ \text{tom to the same units, in} \\ \text{this case sixpences.} \end{array} \right.$$

$$= \frac{1}{9}$$

Note. The answer is just a number and has *no units*. Top and bottom could have been changed to pence, but this would have made the numbers larger and the working heavier.

e.g. Express 660 yd. as a fraction of 1 m.

$$\text{Required fraction} = \frac{660\ \text{yd.}}{1\ \text{m.}}$$

$$= \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{660}}\ \text{yd.}}{\underset{\underset{8}{\cancel{16}}}{\cancel{1760}}\ \text{yd.}}$$

$$= \frac{3}{8}$$

Exercise 7j

Express the first quantity as a fraction of the second.

1.	17*s.* 6*d.*	: £1	**11.**	£1 13*s.*	: 11*s.*
2.	6*s.* 8*d.*	: £1	**12.**	$13\frac{1}{2}$ min.	: $1\frac{1}{2}$ hr.
3.	13*s.* 4*d.*	: £1	**13.**	3*s.* 6*d.*	: 1 guinea
4.	1 ft. 3 in.	: 1 yd.	**14.**	$3\frac{1}{4}$ doz.	: 2 gross.
5.	$\frac{3}{4}$ in.	: 1 ft.	**15.**	$\frac{7}{16}$	: $2\frac{5}{8}$
6.	$7\frac{1}{2}d.$	: 2*s.*	**16.**	1 lb. 2 oz.	: 6 lb. 12 oz.
7.	2*s.* 9*d.*	: 6*s.* 5*d.*	**17.**	£1 3*s.* 4*d.*	: £2 16*s.* 8*d.*
8.	2 ft. 3 in.	: 5 yd.	**18.**	$2\frac{3}{4}d.$	: 1*s.* 10*d.*
9.	$3\frac{1}{2}$ st.	: 2 cwt.	**19.**	2 ft. $1\frac{1}{2}$ in.	: 1 ft. 5 in.
10.	2 ft. 4 in.	: 3 ft. 6 in.	**20.**	$3\frac{1}{2}$ hr.	: $2\frac{1}{3}$ days

PROBLEMS INVOLVING FRACTIONS

(1) From a piece of wood $6\frac{1}{2}$ ft. long, two pieces of length $2\frac{1}{4}$ ft. and $1\frac{2}{3}$ ft. are cut off. What length remains?

$$\text{Length cut off} = 2\frac{1}{4} + 1\frac{2}{3}$$

$$= 3\frac{3+8}{12}$$

$$= 3\frac{11}{12} \text{ ft.}$$

$$\therefore \text{Length remaining} = 6\frac{1}{2} - 3\frac{11}{12}$$

$$= 3\frac{6-11}{12}$$

$$= 2\frac{18-11}{12}$$

$$= 2\frac{7}{12} \text{ ft.}$$

With all problem work it is important to set the work out with clear statements.

(2) A boy had £1 10*s*. He spent £1 2*s*. 6*d*. What fraction of his money was left?

$$\text{Amount left} = £1\ 10s. - £1\ 2s.\ 6d.$$
$$= 7s.\ 6d.$$

$$\text{Required fraction} = \frac{7s.\ 6d.}{£1\ 10s.}$$
$$= \frac{3}{12}$$
$$= \frac{1}{4}$$

Exercise 7k

1. A piece of metal was $3\frac{5}{16}$ in. long. If $1\frac{9}{32}$ in. was cut off, what was the length of the remaining piece?

2. John spends $\frac{1}{3}$ of his pocket money on sweets, $\frac{1}{5}$ towards a school journey and $\frac{3}{10}$ on a football match. He saved the rest towards his holiday. What fraction did he save?

3. Joan had $\frac{1}{2}$ lb. of sweets. Mary had $\frac{3}{5}$ lb. and Pam had $\frac{8}{15}$ lb. Who had the most sweets?

4. Two cars are parked in a road. The first is 13 ft. $7\frac{3}{4}$ in. long and the second is 14 ft. $8\frac{5}{16}$ in. long. If there is a distance of 2 ft. $3\frac{1}{2}$ in. between them, how much road do they occupy?

5. The outside measurements of a picture frame are $16\frac{1}{2}$ in. by $13\frac{1}{8}$ in. If the frame is $\frac{9}{16}$ in. wide, what are its inside measurements?

6. A boy lost $\frac{1}{4}$ of his marbles in a game. If he finished with 21 marbles, how many did he start with?

7. 12 pieces of tape each $3\frac{5}{6}$ in. long are cut from a piece 4 ft. 6 in. long. How much tape was left over?

8. A train travels 108 m. in $2\frac{1}{4}$ hr. What is its average speed?

9. A car travels 209 m. and uses $7\frac{1}{3}$ gal. of petrol. How many miles per gallon is this?

10. A fire burns $\frac{7}{8}$ cwt. of fuel a week. How long will one ton of fuel last? (Give your answer in weeks and days.)

11. Find the cost of $\frac{3}{4}$ cwt. of potatoes at 4*d*. per lb.

12. Find the difference between $3\frac{7}{12}$ and $3\frac{16}{27}$.

13. Taking 1 cu. ft. $= 6\frac{1}{4}$ gal., find:
 (a) What fraction of 1 cu. ft. is 1 gal.
 (b) The volume in cu. ft. of a 40-gal. tank.
 (c) How many gallons can be poured into a tank holding 100 cu. ft.

14. A gardener used $\frac{1}{5}$ of the contents of his water butt one day and in the following day he used $\frac{1}{4}$ of the remainder. He had 36 gal. left in the butt. How much water did he start with?

15. A man left some money to his three sons, the eldest son had $\frac{2}{5}$ of the money, the middle son $\frac{1}{3}$ and the youngest son $\frac{1}{4}$ of the money. The rest was left to charity. Find:
 (a) What fraction of the money was left to charity.
 (b) How much the man left altogether.
 (c) The amounts received by each of his 3 sons.

16. A new car cost £637 10*s*. plus £212 10*s*. purchase tax. What fraction of the makers' price is the purchase tax? What fraction of the total cost is the purchase tax?

8 Decimal Fractions

Look at these three numbers.

2 3 6	(1)
6 2 3	(2)
3 2 6	(3)

Each is formed from the same 3 digits and yet in each number the digit has a different value.

e.g. In (1) the 3 stands for 3 tens or 30.
In (2) the 3 stands for 3 units or 3.
In (3) the 3 stands for 3 hundreds or 300.

Hence the value of each digit depends upon its place. It is this fact that makes this way of writing numbers and working with them a fairly simple affair.

Decimal Fractions use this idea of place value so that working with them is not much different from working with ordinary numbers.

Study this table:

WHOLE NUMBERS				FRACTIONS			DECIMAL FORM
Thousands 1,000	Hundreds 100	Tens 10	Units 1	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	
		3	1	3			31·3
	1	0	3	7			103·7
				3	4		0·34
7	6	1	2	0	4		7612·04
		7	5	6	1	2	75·612

Notice that the decimal point separates the whole number part from the fraction part, and that the number to the left of the decimal point is always the unit.

Exercise 8a

Draw a table like the one above and fit the following numbers into it: 7·23, 4·006, 307, 30·7, 3070, 3·07, 0·0307, 0·307, 41·006, 3721·2.

Changing Decimal Fractions into Vulgar Fractions

Look at the table above.

$$31{\cdot}6 = 31\frac{6}{10} \text{ or } \frac{316}{10}$$

$$103{\cdot}7 = 103\frac{7}{10} \text{ or } \frac{1037}{10}$$

$$0{\cdot}34 = \frac{\overset{17}{\cancel{34}}}{\underset{50}{\cancel{100}}} = \frac{17}{50}$$

$$7612{\cdot}04 = 7612\frac{4}{100} \text{ or } 7612\frac{1}{25}$$

Further:

$$0{\cdot}125 = \frac{\overset{1}{\cancel{125}}}{\underset{8}{\cancel{1000}}} = \frac{1}{8}$$

and

$$17{\cdot}65 = 17\frac{\overset{13}{\cancel{65}}}{\underset{20}{\cancel{100}}} = 17\frac{13}{20}$$

Changing Vulgar Fractions into Decimal Fractions

$$\frac{37}{100} = 0{\cdot}37$$

$$13\frac{7}{10} = 13{\cdot}7$$

A harder example: By expressing the fraction $\frac{7}{25}$ with a suitable denominator, put it in decimal form.

By the Golden Rule of Fractions $\frac{7}{25} = \frac{7}{25} \times \frac{4}{4}$

$$= \frac{28}{100}$$

$$= 0{\cdot}28$$

Similarly $\frac{3}{4} = \frac{3}{4} \times \frac{25}{25}$

$$= \frac{75}{100}$$

$$= 0{\cdot}75$$

Multiplying Decimal Fractions by 10 and Multiples of 10

e.g. $$0{\cdot}73 \times 10 = \frac{73}{\underset{10}{\cancel{100}}} \times \overset{1}{\cancel{10}} = 7{\cdot}3$$

$$3{\cdot}87 \times 100 = \frac{387}{100} \times 100 = 387$$

Hence if a decimal fraction is multiplied by ten the digits move one place to the left of the decimal point which remains fixed. (Often for memory purposes the principle is expressed as follows: Multiplication by 10 means move decimal point one to the right. This is incorrect because this changes the value of each digit by moving one place to the left.)
Further as the multiplication is by 100, 1,000 etc., then the digits are moved 2, 3 etc. to the left. The number of places moved is equal to the number of zero digits in the multiplier.

Dividing Decimal Fractions by 10 and Multiples of 10

e.g. $$6{\cdot}8 \div 10 = \frac{68}{10} \times \frac{1}{10} = \frac{68}{100} = 0{\cdot}68$$

$$0{\cdot}07 \div 100 = \frac{7}{100} \times \frac{1}{100} = \frac{7}{10000} = 0{\cdot}0007$$

Hence if a decimal fraction is divided by 10, 100, 1,000 etc. the digits are moved 1, 2, 3 etc. places to the right. The number of places moved is equal to the number of zero digits in the divisor.

Exercise 8b

1. Change the following into vulgar fractions in their lowest terms:

(a) 2·6 (b) 0·37 (c) 0·08 (d) 18·5 (e) 10·04 (f) 0·625 (g) 4·35 (h) 1·0625 (i) 0·0075 (j) 0·137

2. Change the following into decimal fractions by putting, where necessary, the denominators into multiples of 10.

(a) $3\frac{7}{10}$ (b) $4\frac{3}{100}$ (c) $\frac{17}{1000}$ (d) $\frac{1}{4}$ (e) $\frac{3}{5}$ (f) $1\frac{17}{20}$ (g) $\frac{19}{25}$ (h) $\frac{47}{50}$ (i) $\frac{3}{125}$ (j) $2\frac{1}{8}$ (k) $3\frac{5}{8}$ (l) $\frac{273}{50}$ (m) $\frac{313}{200}$ (n) $\frac{178}{25}$ (o) $\frac{21}{500}$

3. Write down the answers to the following:

(a)	3·6	×	10	(k)	13·07	×	10
(b)	3·6	×	100	(l)	13·07	×	1000
(c)	3·6	×	1000	(m)	13·07	÷	10
(d)	3·6	÷	10	(n)	13·07	÷	100
(e)	3·6	÷	100	(o)	13·07	÷	10000
(f)	3·6	÷	1000	(p)	620	÷	10
(g)	0·03	×	100	(q)	620	÷	100
(h)	0·03	÷	10	(r)	0·0017	×	100
(i)	0·03	÷	1000	(s)	0·0017	×	1000
(j)	0·03	×	10	(t)	10·06	÷	1000

Addition of Decimal Fractions

Add the numbers 3 + 176 + 30 + 4307. Unless you are very expert, it is necessary to write these numbers down as follows:

Thousands	Hundreds	Tens	Units
			3
	1	7	6
		3	0
4	3	0	7
4	5	1	6

Notice that the units are under each other.

When adding decimal fractions the same rule must be observed.
e.g. 3·6 + 13·75 + 0·03 + 14·2

Tens	Units	·	Tenths	Hundredths
	3	·	6	
1	3	·	7	5
	0	·	0	3
1	4	·	2	
3	1	·	5	8

Again the units are under each other or, in other words, the decimal points are under each other.

Subtraction of Decimal Fractions

Here the same rule applies: place the units (or decimal points) under each other.

e.g. 3·76 — 1·307

$$\begin{array}{r} 3{\cdot}760 \\ \underline{1{\cdot}307} \\ 2{\cdot}453 \end{array}$$

Fill in the empty space with a zero.

or, what is the difference between 0·893 and 1·6?

$$\begin{array}{r} 1{\cdot}600 \\ \underline{0{\cdot}893} \\ 0{\cdot}707 \end{array}$$

Be careful here to make sure that you are taking the smaller number from the bigger one.

Exercise 8c

1. **(a)** Add
$$\begin{array}{r} 2{\cdot}3 \\ 3{\cdot}67 \\ 14{\cdot}2 \\ \underline{0{\cdot}68} \end{array}$$

(b) Add
$$\begin{array}{r} 14{\cdot}06 \\ 30{\cdot}2 \\ 1{\cdot}793 \\ \underline{2{\cdot}8} \end{array}$$

(c) Add
$$\begin{array}{r} 136{\cdot}9 \\ 21{\cdot}24 \\ 100{\cdot}06 \\ \underline{13{\cdot}923} \end{array}$$

(d) Add
$$\begin{array}{r} 12{\cdot}807 \\ 0{\cdot}006 \\ 19{\cdot}24 \\ 1{\cdot}97 \\ \underline{13{\cdot}862} \end{array}$$

(e) Add
$$\begin{array}{r} 14{\cdot}872 \\ 1{\cdot}998 \\ 327{\cdot}62 \\ 136{\cdot}0 \\ \underline{14{\cdot}532} \end{array}$$

2. (a) Subtract $\begin{array}{r} 14{\cdot}87 \\ \underline{3{\cdot}26} \end{array}$ (b) Subtract $\begin{array}{r} 91{\cdot}26 \\ \underline{13{\cdot}8} \end{array}$ (c) Subtract $\begin{array}{l} 1{\cdot}49 \\ \underline{0{\cdot}368} \end{array}$

(d) Subtract $\begin{array}{r} 13{\cdot}0 \\ \underline{4{\cdot}987} \end{array}$ (e) Subtract $\begin{array}{r} 17{\cdot}662 \\ \underline{1{\cdot}9987} \end{array}$

3. (a) $3{\cdot}06 + 1{\cdot}23 + 7{\cdot}987 + 0{\cdot}332$
   (b) $17{\cdot}9 + 0{\cdot}037 + 14{\cdot}62 + 300{\cdot}9 + 1{\cdot}76$
   (c) $1934{\cdot}2 + 400{\cdot}18 + 17{\cdot}623 + 49 + 308{\cdot}9$
   (d) $0{\cdot}006 + 3{\cdot}984 + 0{\cdot}873 + 8{\cdot}99 + 1{\cdot}2$
   (e) $6{\cdot}337 + 4 + 492{\cdot}8 + 32{\cdot}9 + 98{\cdot}87 + 600$

4. (a) $13{\cdot}2 - 9{\cdot}78$
   (b) $12{\cdot}224 - 8{\cdot}795$
   (c) Find the difference between 3·9 and 7·6.
   (d) Subtract 14·37 from 28·2.
   (e) Take 0·899 from 1·2.

5. (a) $(9{\cdot}637 + 0{\cdot}438 + 2{\cdot}61) - (1{\cdot}432 + 0{\cdot}88)$
   (b) $693{\cdot}7 - (40{\cdot}61 + 13{\cdot}8 + 0{\cdot}834)$
   (c) $93{\cdot}26 - 14{\cdot}8 + 79{\cdot}6 + 3{\cdot}78 - 31{\cdot}038$
   (d) $12{\cdot}78 + 49{\cdot}2 - 14{\cdot}637 + 6 - 9{\cdot}81$
   (e) $14{\cdot}73 + 297{\cdot}2 - 16{\cdot}8 - 49{\cdot}337 + 14$

Multiplication of Decimal Fractions

Study these examples carefully.

(a) $2{\cdot}1 \times 11 = \frac{21}{10} \times 11 = \frac{231}{10} = 23{\cdot}1$

(b) $21 \times 1{\cdot}1 = 21 \times \frac{11}{10} = \frac{231}{10} = 23{\cdot}1$

(c) $2{\cdot}1 \times 1{\cdot}1 = \frac{21}{10} \times \frac{11}{10} = \frac{231}{100} = 2{\cdot}31$

(d) $0{\cdot}21 \times 1{\cdot}1 = \frac{21}{100} \times \frac{11}{10} = \frac{231}{1000} = 0{\cdot}231$

(e) $2{\cdot}1 \times 0{\cdot}11 = \frac{21}{10} \times \frac{11}{100} = \frac{231}{1000} = 0{\cdot}231$

(f) $0{\cdot}21 \times 0{\cdot}11 = \frac{21}{100} \times \frac{11}{100} = \frac{231}{10000} = 0{\cdot}0231$

Consider example (a). 2·1 × 11 contains one figure after the decimal point. How many figures does the answer 23·1 contain after the decimal point?

Now look through the other examples, counting the number of 'decimal places' in both numbers in the left-hand column and then the number of 'decimal places' in the answer. What do you notice?

This leads to a simple way of multiplying numbers containing decimal fractions.

EXAMPLES

(a) 2·31 × 7 = 16·17

Working:
$$\begin{array}{r} 231\ \times \\ 7 \\ \hline 1617 \end{array}$$

Multiply 231 × 7 as ordinary numbers. Count up the number of 'decimal places' in 2·31 × 7, i.e. two. Count this number in the answer starting from the right.

(b) 3·62 × 2·4 = 8·688

Working:
$$\left.\begin{array}{r} 362\ \times \\ 24 \end{array}\right\} \text{3 'decimal places' altogether.}$$
$$\begin{array}{r} 7240 \\ 1448 \\ \hline 8688 \end{array}$$

Now count 3 'decimal places' from the right of the answer.

(c) 0·1083 × 0·07 = 0·007581

Working:
$$\left.\begin{array}{r} 1083\ \times \\ 7 \end{array}\right\} \text{6 'decimal places' altogether.}$$
$$\begin{array}{r} \hline 7581 \end{array}$$

In this case two 'zeros' must be placed before the 7 to make up the right number of places.

Exercise 8d

1. 3·6 × 7
2. 3·6 × 0·7
3. 3·6 × 70
4. 0·36 × 7
5. 0·36 × 0·7
6. 3·6 × 0·7
7. 0·36 × 0·07
8. 360 × 0·007
9. 0·036 × 0·7
10. 36 × 0·7
11. 3·6 × 1·9
12. 47 × 23·1
13. 23·6 × 2·7
14. 9·81 × 0·32
15. 461 × 0·048
16. 0·99 × 0·61
17. 3·73 × 0·042
18. 1·27 × 1·36
19. 0·807 × 2·9
20. 0·601 × 0·98

Division of Decimal Fractions

$\frac{0\cdot8}{4} = 0\cdot2$ This is correct because $0\cdot2 \times 4 = 0\cdot8$

$\frac{0\cdot8}{0\cdot4} = 2$ This is correct because $0\cdot4 \times 2 = 0\cdot8$

With simple examples like these the answer can always be easily checked. In harder examples the division of the actual numbers is not difficult, but it is important to find the position of the decimal point in the answer.

e.g. $\frac{1\cdot08}{1\cdot2} = \frac{10\cdot8}{12} = 0\cdot9$

Make the divisor (in this case 1·2) a whole number: 1·2 becomes 12 when multiplied by 10. Now apply the Golden Rule of Fractions and multiply the top number by 10, thus not changing the value of the answer.

Working:

$$\begin{array}{r} 0\cdot9 \\ 12\overline{)10\cdot8} \end{array}$$

Provided the divisor is a whole number the decimal point in the answer will be exactly above the decimal point in the dividend (in this case 10·8).

Here is a harder example:

$\frac{0\cdot8568}{0\cdot28} = \frac{85\cdot68}{28}$ (Make divisor a whole number, by multiplying top and bottom by 100.)

$= 3\cdot06$ Leave this space for the answer.

Working:

$$\begin{array}{r} 3\cdot06 \\ 28\overline{)85\cdot68} \\ 84 \\ \hline 168 \\ 168 \\ \hline \end{array}$$

Exercise 8e

Set these examples out as shown above.

By short division

1. $4\cdot9 \div 0\cdot7$
2. $12\cdot8 \div 0\cdot8$
3. $0\cdot128 \div 0\cdot08$
4. $1\cdot28 \div 80$
5. $18\cdot54 \div 0\cdot6$
6. $7 \div 8$
7. $4 \div 50$
8. $0\cdot0297 \div 1\cdot1$
9. $0\cdot063324 \div 0\cdot09$
10. $4\cdot512 \div 0\cdot12$

By long division

11. $8 \cdot 28 \div 3 \cdot 6$
12. $0 \cdot 1768 \div 0 \cdot 17$
13. $28 \cdot 897 \div 3 \cdot 7$
14. $21 \div 1 \cdot 4$
15. $0 \cdot 6732 \div 0 \cdot 0017$
16. $3219 - 8 \cdot 7$
17. $0 \cdot 0038294 \div 0 \cdot 82$
18. $3 \div 64$
19. $3 \cdot 4151 \div 0 \cdot 923$
20. $78 \cdot 3648 \div 8 \cdot 64$

An Important Task

$\frac{1}{8} =$ $\frac{5}{8} =$

$\frac{1}{4} =$ $\frac{3}{4} =$

$\frac{3}{8} =$ $\frac{7}{8} =$

$\frac{1}{2} =$

Treat each of these examples as a division. E.g. $\frac{1}{8} = 0 \cdot 125$

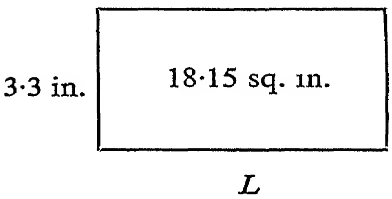
$$8 \,\underline{|1 \cdot 000}$$
$$0 \cdot 125$$

Make a table of your answers and learn them.

Problems involving Decimal Fractions

As with all problems it is important to begin each stage of your answer with a clear statement. Keep the working separate from the rest.
Here is an example:

A postcard is 3·3 in. wide. Its area is 18·15 sq. in. What is its length?



Area = Length × Breadth

$\therefore A = LB$

or $L = \dfrac{A}{B}$

$= \dfrac{18 \cdot 15}{3 \cdot 3}$

$= \dfrac{181 \cdot 5}{33}$

$= 5 \cdot 5$ in.

Working:

$$\begin{array}{r} 5 \cdot 5 \\ 33\,\overline{)181 \cdot 5} \\ \underline{165} \\ 16\,5 \\ \underline{16\,5} \end{array}$$

Exercise 8f

1. A piece of card is 7·8 in. long. If a piece 3·62 in. is cut off, what length remains?

2. Two strips of metal of lengths 3·76 and 4·87 in. are welded together. How long is the resulting strip of metal?

3. What is the difference between 0·936 and 1·01?

4. By changing them into decimal fractions, where required, arrange the following mixed numbers in order of size, starting with the smallest.

$$3\tfrac{3}{4},\ 3{\cdot}761,\ 3\tfrac{5}{8},\ 3\tfrac{13}{16}$$

5. A book containing 250 leaves is 0·8 in. thick. What is the thickness of the paper?

6. A piece of metal was supposed to be cut to a length of 4·5 in. Its actual length was 4·468 in. How much too short was it?

7. How many pieces of wire 2·4 in. long can be cut from a piece 17 in. in length? What length remains?

8. A boy runs 100 yd. on 5 occasions in the following times: 12·2 sec., 12·4 sec., 11·8 sec., 11·95 sec., 12·3 sec. What is his average time over this distance?

9. A train does a journey of 213 m. in 4 hr. What is its average speed, expressed in decimal form? How many miles would it travel in 7 hr. at the same average speed?

10. A water cistern holds 7·3 cu. ft. of water. Find the weight of water in the cistern if 1 cu. ft. of water weighs 62·3 lb.

11. In France petrol is bought in litres. If 1 litre = 1·76 pints, how many litres of petrol are required to fill the petrol tank of a car if it usually holds 7 gal.? Give your answer as a whole number.

12. A rectangle is 7·8 in. long and 3·45 in. wide. Find its area and its perimeter.

13. The average rainfall over a period of 31 days was 0·24 in. What was the total rainfall over the period?

14. Glycerine weighs 78·6 lb. per cu. ft. and methylated spirit weighs 51·2 lb. per cu. ft. What is the total weight of 7 cu. ft. of glycerine and 11 cu. ft. of methylated spirit?

15. The total wind pressure on the sail of a boat is 116 lb. wt. If the sail has an area of 135 sq. ft., what is the pressure per square foot?

9 Percentages

Here is the family tree of the Fraction family.

MR and MRS FRACTION

VULGAR FRACTION   DECIMAL FRACTION   PERCENTAGE

There is another member of the family, RATIO. We shall meet Ratio in Book II.

A percentage is a fraction written in a special way. It is often used in everyday life.

$$10 \text{ per cent (written } 10\%) \text{ or } \frac{10}{100} \begin{matrix}\text{ten}\\ \text{per}\\ \text{cent}\end{matrix}$$

$$\therefore\ 10\% = \frac{1}{10} \text{ after cancelling}$$

$$\text{Further}\quad 12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100}$$

$$= \frac{12\frac{1}{2} \times 2}{100 \times 2}$$

Multiply top and bottom by 2 to clear the top of fractions.

$$= \frac{\overset{1}{\cancel{25}}}{\underset{8}{\cancel{200}}}$$

$$= \frac{1}{8}$$

Exercise 9a

Change the following percentages into vulgar fractions.

1. 50%
2. 25%
3. 75%
4. $62\frac{1}{2}$%
5. $37\frac{1}{2}$%
6. 5%
7. 160%
8. 65%
9. 250%
10. $33\frac{1}{3}$%
11. $83\frac{1}{3}$%
12. $31\frac{1}{4}$%

To Find a Percentage of a Given Quantity

e.g. What is 30% of £2?

First we change the percentage to a fraction.

$$30\% \text{ of } £2 = \frac{3\cancel{0}}{10\cancel{0}\cancel{0}} \times 40s. \qquad \text{Replace the 'of' by '}\times\text{' and change the pounds to shillings.}$$

$$= 12s.$$

Also, find 45% of £3 10*s*.

$$45\% \text{ of } £3\ 10s. = \frac{\overset{9}{\cancel{45}}}{\underset{2}{\cancel{100}}} \times 7\cancel{0}s.$$

$$= \frac{63}{2}s.$$

$$= 31\tfrac{1}{2}s.$$

$$= £1\ 11s.\ 6d.$$

Exercise 9b

Calculate:

1.	15% of £1.	(Ans. in *s*.)
2.	75% of 3 lb.	(Ans. in oz.)
3.	20% of 15 ft.	(Ans. in ft.)
4.	$12\frac{1}{2}$% of 1 cwt.	(Ans. in lb.)
5.	30% of £50.	
6.	$66\frac{2}{3}$% of 4 yd.	(Ans. in ft.)
7.	$62\frac{1}{2}$% of 1 m.	(Ans. in yd.)
8.	90% of 1 ton.	(Ans. in cwt.)
9.	140% of £1 15*s*.	(Ans. in £ *s*.)
10.	$41\frac{2}{3}$% of a day.	(Ans. in hr.)

Changing Vulgar and Decimal Fractions into Percentages

To change $\frac{3}{4}$ into a percentage it is necessary to make the bottom number 100.

By the Golden Rule of Fractions

$$\frac{3}{4} = \frac{\frac{3}{4} \times 100}{100} \qquad \text{(A)}$$

$$= \frac{3}{\underset{1}{\cancel{4}}} \times \overset{25}{\cancel{100}}\ \%$$

$$= 75\%$$

Further:

$$\frac{7}{8} = \frac{7}{\cancel{8}_{\,2}} \times \cancel{100}^{\,25}\ \% \quad \text{Leaving out the step marked (A) above.}$$

$$= \frac{175}{2}\%$$

$$= 87\tfrac{1}{2}\%$$

In the same way, a decimal fraction can be converted into a percentage.

$$\begin{aligned} 0{\cdot}64 &= 0{\cdot}64 \times 100\% \\ &= 64\% \end{aligned}$$

Exercise 9c

Express the following fractions as percentages:

1. $\frac{3}{8}$
2. $\frac{1}{3}$
3. $1\frac{3}{4}$
4. $\frac{1}{6}$
5. $\frac{37}{50}$
6. $\frac{2}{7}$
7. $0{\cdot}37$
8. $1{\cdot}9$
9. $0{\cdot}362$
10. $\frac{5}{16}$
11. $\frac{17}{25}$
12. $\frac{19}{75}$

Expressing One Quantity as a Percentage of Another

This process requires one quantity to be expressed as a fraction of the other. The resulting fraction is then changed into a percentage.

e.g. Express 1*s*. 6*d*. as a percentage of 2*s*. 6*d*.

$$\text{Required percentage} = \frac{1s.\ 6d.}{2s.\ 6d.} \times 100\%$$

$$= \frac{3}{\cancel{5}_{\,1}} \times \cancel{100}^{\,20}\ \% \text{ (changing into sixpences)}$$

$$= 60\%$$

Exercise 9d

Express the first quantity as a percentage of the second.

1. 2*s*. 6*d*. : 10*s*.

2. 6*s*. 8*d*. : £1.

3. 1 ft. : 7 ft. 6 in.

4. 19*s*. : £2.

5. 36 sec. : 3 min.

6. 42 lb. : 1 cwt.

7. 48 sq. in. : 1 sq. ft.

8. £12 : £10.

9. 7*s*. : 3 guineas.

10. 1*s*. 3*d*. : 4*s*. 7*d*.

An Important Example

In a school with 960 children, 55% of the children are girls.

(a) What percentage of the children are boys?

(b) How many boys are there?

(a) The percentage of boys $= 100\% - 55\%$ (because the whole school requires 100%)

$= 45\%$

(b) The number of boys $= 45\%$ of 960

$$= \frac{\overset{9}{\cancel{45}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{100}}} \times \overset{48}{\cancel{960}}$$

$$= 9 \times 48$$

$$= 432$$

Exercise 9e

1. The customs duty on an article is $33\frac{1}{3}\%$ of its value. What is the duty on an article worth £2?

2. Find 5% of 480.

3. A potato is 95% water. How much water is there in a ton of potatoes?

4. A man dies and leaves a sum of money. He leaves 12% of it to charity, 50% to his wife and the rest to his son. What percentage of the money is received by his son?

5. In a certain examination 45% of the candidates fail. What percentage of the candidates pass? If 360 candidates took the examination, how many passed?

6. A boy got 17 examples correct out of 25 in his homework. His sister got 15 right out of 20 in hers. Express both marks as percentages and say whether the boy or his sister got the best mark.

7. A line is 10 in. long. A boy measured it and gave its length as $9\frac{3}{4}$ in. Express his error as a percentage of the correct length.

8. A shopkeeper buys a penknife for 2*s*. 6*d*. and makes 20% profit. What profit does he make, and at what price does he sell the knife?

9. In a crate containing 1 gross tins of fruit, 30 were damaged. What percentage were undamaged?

10. A man pays 6% of his salary towards a pension fund. If he earned £1,250 in a year, how much did he pay to the fund?

11. If 27% of a number is 135, what is the number?

12. The population of a town increased by 5% in a year. If the population was 80,000 at the beginning of the year, how much was it at the end of the year?

10 Mathematical Diagrams

Pictorial Mathematics or Graphs

When you are ill, especially if you are in hospital, your temperature is taken twice a day and these readings are recorded on a chart thus:

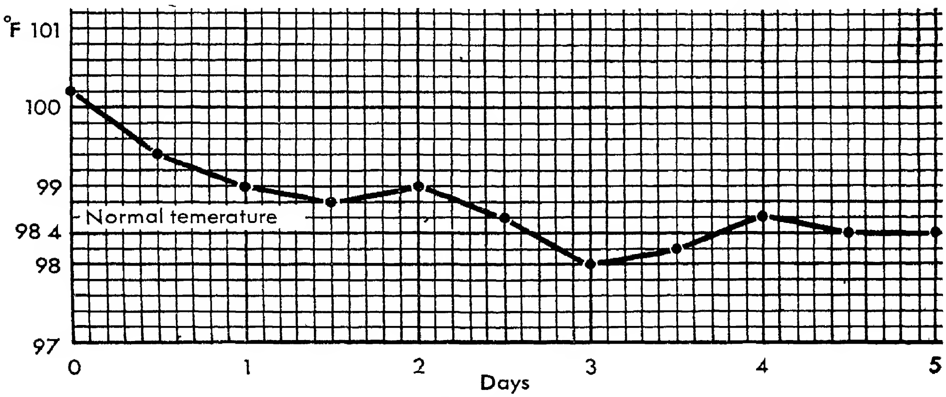


fig. 10.1

With the help of this chart the doctor is able to see *at a glance* how you are progressing. Take particular notice of the words in italics in the last sentence, for this is exactly the purpose for which these mathematical diagrams are designed.

Here is an example which shows this principle very clearly. Each year when the Chancellor of the Exchequer presents his Budget to Parliament he has to explain how the money he proposes to raise in taxes is to be spent. In one year he estimated that each pound would be spent as follows:

Army, Navy and Air Force	7	6
Interest on National Savings, etc.	2	10
Health Services	1	10
Education	1	4
Family Allowances, Pensions, etc.	1	10
Food Subsidies	1	0
Housing, Police, Roads, etc.	1	1
General Services	2	1
Surplus		6
£1	0	0

If you look at these figures it is difficult to see them all at once and almost impossible to compare one item with another.
We can, however, construct a diagram which will show us the complete picture, especially the relationship between one item and another at a glance.

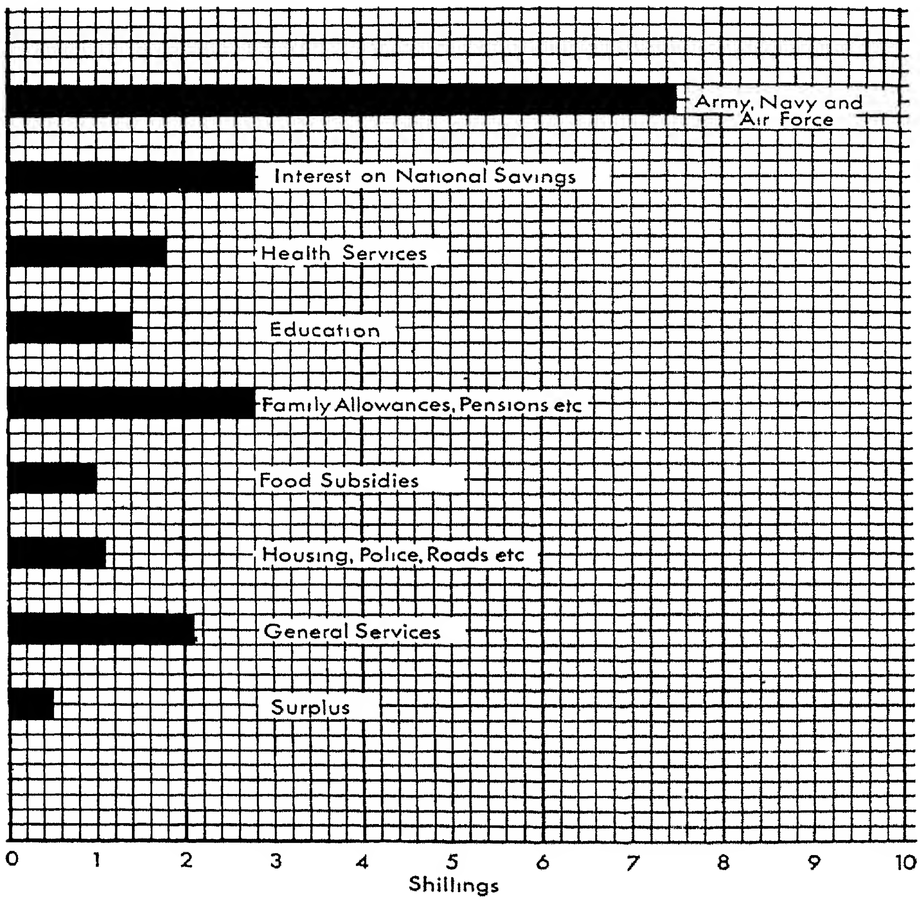


fig. 10.2

This diagram is called a Bar Graph. You will notice that it is drawn on squared paper, a practice which should always be observed when drawing any kind of graph, as it is then much easier to construct suitable scales to represent the various amounts.

There is a third kind which is sometimes used. Consider the following example.

A man earns £20 per week. He spends one-third of his income on rent, rates, etc., one-quarter on food, one-eighth on clothing, he saves one-sixth and he spends the remainder on pleasure. Exhibit these figures on a graph and find what fraction of his income he spends on pleasure.

For this type of problem it is best to draw what we call a Pie-Chart. You will see the reason for this name when we have completed the diagram. First draw a circle of two-inch radius. You will know from your geometry that any circle contains 360 degrees. One-third of 360° is $\frac{360}{3} = 120°$. Using your protractor mark off an angle of 120°. Join the two points on the circumference to the centre of the circle. This slice represents the amount paid in rent and rates. One-quarter of 360° is $\frac{360}{4} = 90°$. Mark off another 90°, join this point to the centre and this slice represents the amount paid for food. Similarly one-eighth of 360° is 45° and one-sixth of 360° is 60°, these two slices representing the amounts spent on clothing and the amount saved respectively.

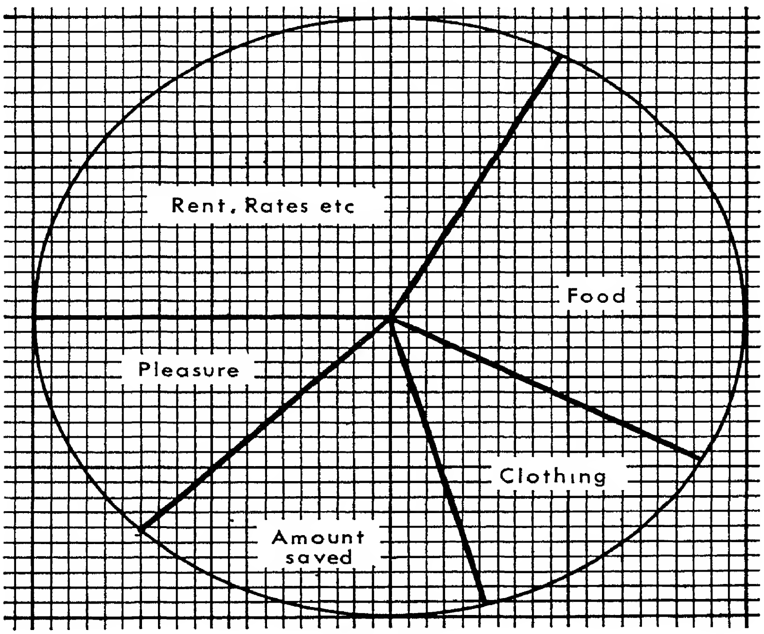


fig. 10.3

Adding all these angles together we get

$$120 + 90 + 45 + 60 = 315$$

Subtracting from 360 we get

$$360 - 315 = 45$$

and

$$\frac{45}{360} = \frac{1}{8}$$

Thus he spends one-eighth of his income on pleasure.
We can check this by adding fractions

$$\frac{1}{3} + \frac{1}{4} + \frac{1}{8} + \frac{1}{6} = \frac{8 + 6 + 3 + 4}{24} = \frac{21}{24}$$

Hence remainder

$$= \frac{24 - 21}{24} = \frac{3}{24} = \frac{1}{8}$$

We can further check our result by cutting a slice equal in size to the slice on our diagram from another piece of paper and testing that this is exactly one-eighth of the complete circle.

Exercise 10a

NOTE. Line and Column (or Bar) Graphs should always be drawn on squared paper, preferably on paper divided into one-inch squares with each square divided into 100 small squares. Pie-charts may also be drawn on squared paper.

1. The maximum temperatures for the first ten days of April 1959 are shown in the following table. Exhibit these figures on a line graph.

April	Day	1	2	3	4	5	6	7	8	9	10
1959	Max. Temp. (°F)	58	59	63	63	55	54	51	56	51	53

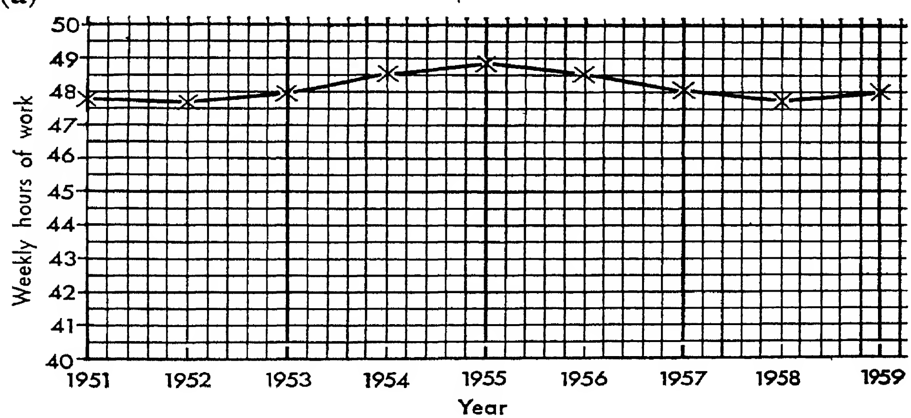
2. Make a table of the heights of all the pupils in your class and exhibit these on a line graph. Make a similar table of weights and draw a suitable graph of these figures.

3. The following table shows the average number of hours worked per week and the average weekly wages in shillings earned during the years 1951 to 1959.

YEAR	MEN		WOMEN		ALL WORKERS	
	HOURS	WAGES	HOURS	WAGES	HOURS	WAGES
1951	47·8	169	41·5	90	46·1	141
1952	47·7	179	41·8	96	46·1	152
1953	47·9	189	42·0	102	46·3	160
1954	48·5	204	41·9	108	46·7	172
1955	48·9	223	41·8	115	46·9	187
1956	48·5	238	41·5	123	46·6	201
1957	48·2	252	41·2	130	46·4	212
1958	47·7	257	41·2	134	46·0	217
1959	48·0	263	41·5	137	46·3	223

Three graphs, shown below, have been drawn showing the relationship between (a) the year and the average number of hours worked by men, (b) the year and the average number of hours worked by women, (c) the year and the average number of hours worked by all workers, and these have all been combined into one graph in order to see at a glance a comparison between these values. Study these graphs carefully, noting especially how the frames (or axes) have been selected and then, using the table above, draw four similar graphs to show the relationships between the year and the average wages earned by the various groups.

(a)



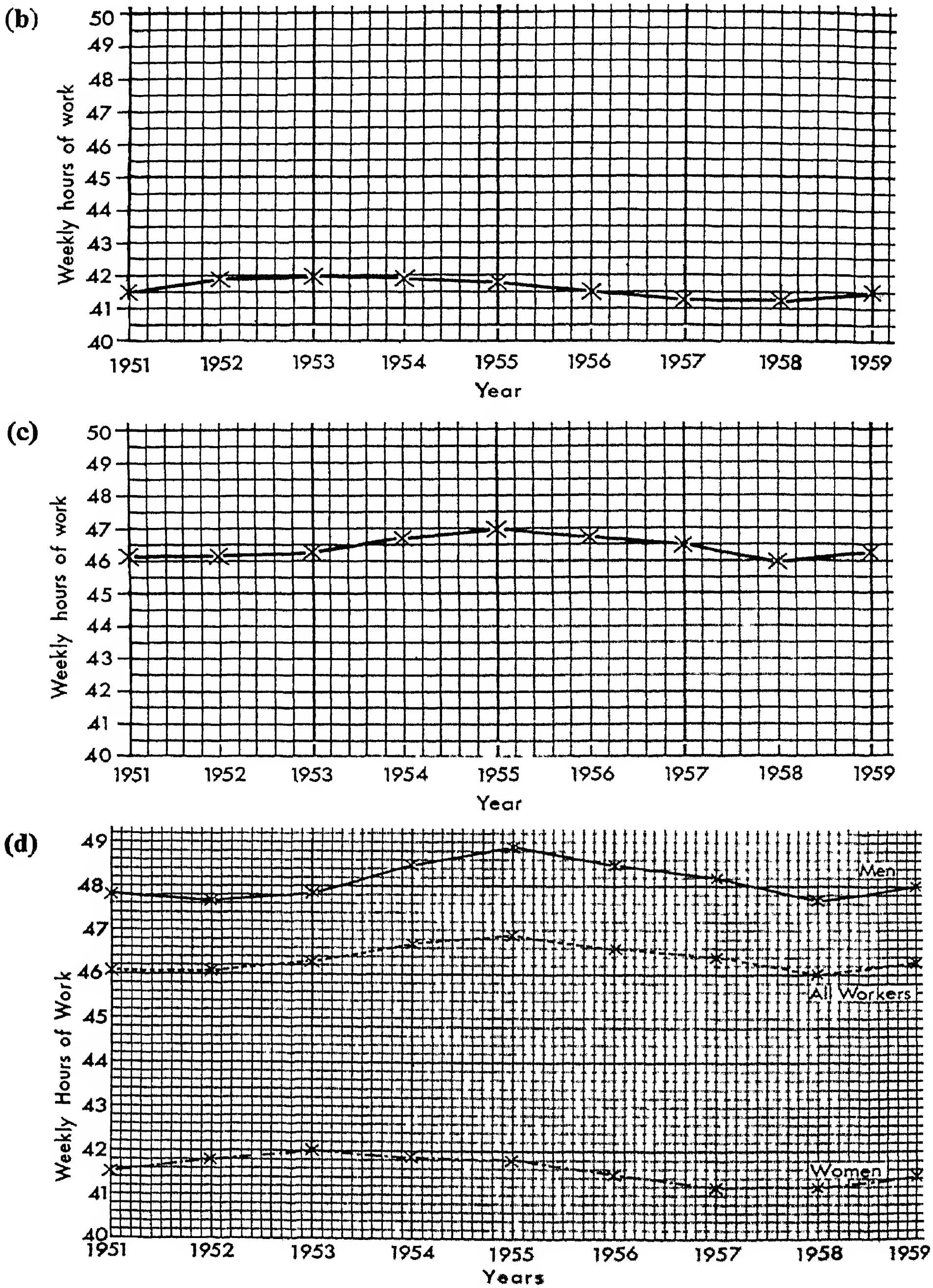

(b)
Weekly hours of work
50
49
48
47
46
45
44
43
42
41
40
1951
1952
1953
1954
1955
1956
1957
1958
1959
Year
(c)
Weekly hours of work
Year
(d)
Weekly Hours of Work
Men
All Workers
Women
Years

fig. 10.4

4. The following table shows the details of fatal accidents on the roads to children and teenagers during the year 1959. Using the same axes draw five line graphs to exhibit these figures. Find from your graph: **(a)** the total number of children aged 14 who were killed during 1959, **(b)** the total number of pedal cyclists of all ages who were killed during 1959, **(c)** the total number of young people under 19 years of age who were killed on the roads in 1959.

Age in yr.	under 1	1	2	3	4	5	6	7	8	9
Pedestrians	3	38	58	57	53	51	43	35	28	20
Pedal cyclists			3	3	1	2	2	4	7	10
All passengers	5	12	7	8	2	3	7	5	0	4
Drivers										
Motor cyclists										

Age in yr.	10	11	12	13	14	15	16	17	18	19
Pedestrians	21	20	22	9	7	6	15	11	7	9
Pedal cyclists	9	25	24	32	24	31	25	13	8	15
All passengers	5	1	5	1	2	19	25	48	51	52
Drivers							1	1	8	20
Motor cyclists					1	0	50	87	88	111

5. During the rainy season 1960 some fields in the Thames Valley were flooded. During the first day 2 acres were flooded and the second day the flood spread over 3 acres more. During the following five days 4·3 acres, 4·8 acres, 3·6 acres, 2·3 acres, 0·7 acre were added in succession. Draw a column graph showing the total area flooded during each day of the week.

6. The Chancellor of the Exchequer estimated that he would raise each pound of the National Revenue in the following way:

Income tax and Surtax	5	5	Purchase tax	1	2
Company tax	4	9	Entertainments tax		4
Death duties		9	Motor duties and Petrol	1	8
Tobacco tax	2	10	Stamp duties, etc.		9
Alcohol	1	8	Broadcast licences, etc.		8

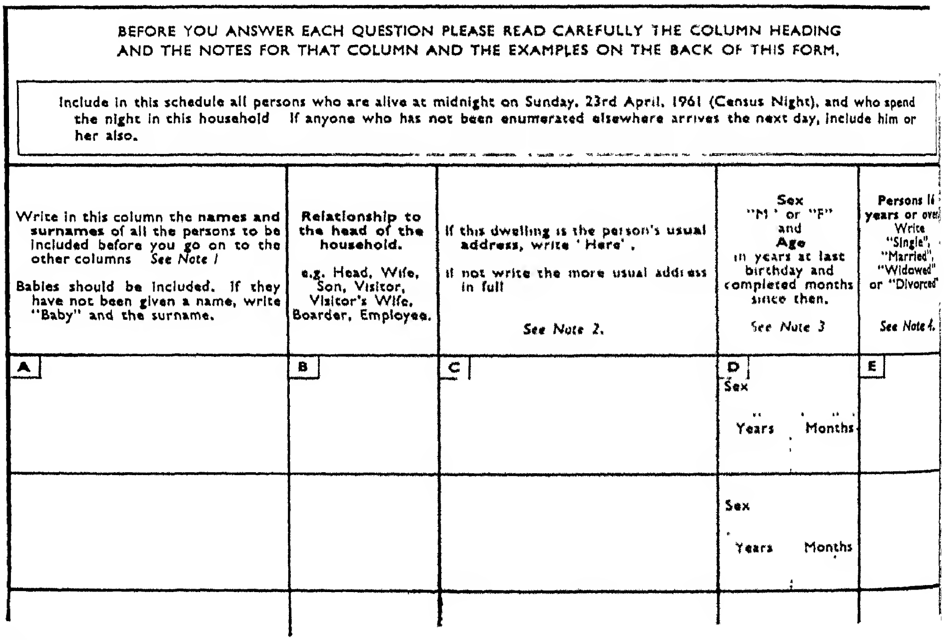
Exhibit these figures on a bar graph.

7. In the manufacture of a certain article it is calculated that one-half of the selling price is the cost of labour required, one-quarter of

the selling price the cost of materials, one-eighth the cost of advertising and marketing, and the remainder represents the profit made by the manufacturer. Represent these figures on a pie-chart. What fraction of the selling price is the manufacturer's profit and, if each article costs 15*s*., how much profit does he make if he sells 144 articles?

8. The following figures show the weight of fish (to the nearest 100,000 cwt.) caught during a year around the coasts of Great Britain. Plot these figures on a pie-chart. (*Hint.* To ease your working take the figures as 6·0 times 100,000, 1·2 times 100,000, etc.)

KINDS OF FISH	WEIGHT CAUGHT (cwt.)
Cod	6,000,000
Haddock	1,000,000
Hake	200,000
Plaice	600,000
Whiting	200,000
Herring	1,200,000
Mackerel	200,000
Other fish	2,000,000



BEFORE YOU ANSWER EACH QUESTION PLEASE READ CAREFULLY THE COLUMN HEADING AND THE NOTES FOR THAT COLUMN AND THE EXAMPLES ON THE BACK OF THIS FORM.

Include in this schedule all persons who are alive at midnight on Sunday, 23rd April, 1961 (Census Night), and who spend the night in this household. If anyone who has not been enumerated elsewhere arrives the next day, include him or her also.

Write in this column the names and surnames of all the persons to be included before you go on to the other columns *See Note 1* Babies should be included. If they have not been given a name, write "Baby" and the surname.	Relationship to the head of the household. e.g. Head, Wife, Son, Visitor, Visitor's Wife, Boarder, Employee.	If this dwelling is the person's usual address, write 'Here'. If not write the more usual address in full *See Note 2.*	Sex "M" or "F" and Age in years at last birthday and completed months since then. *See Note 3*	Persons 16 years or over Write "Single", "Married", "Widowed" or "Divorced" *See Note 4.*
A	B	C	D Sex Years Months	E
			Sex Years Months	

All the graphs which you have drawn are called *Graphs of Statistics.* Statistics are defined as being 'numerical facts systematically collected'. Thus to get a complete picture of the whole process we must examine how these figures are collected.

Every ten years it is necessary for the Registrar General to know exactly how many men, women and children are living in each house in the country. A National Census Form is therefore delivered to each house and on a certain fixed day your father or mother is required to complete this and return it. To make sure that the information is correct a Census official calls on each householder to help him complete the form and also the questions are framed in such a way that either *a definite number or a simple 'Yes' or 'No' is required.* In other words the form must be made as simple as possible. Here is an example of the type of form which was circulated in the recent census.

This 'Question Form', sometimes called a 'Questionnaire', is of necessity rather complicated, but we can design much simpler forms for a variety of purposes.

PLEASE WRITE IN INK

All married, widowed or divorced women. Write at (i) the total number of children born alive to her in marriage *See Note 5* Were any of these children born after 23rd April, 1960? Write "Yes" or "No" at (ii)	**All married women** Write at (i) the date of her present marriage. Has she been married more than once? Write "Yes" or 'No" at (ii) If "Yes" fill in column **H**	**Widowed or divorced women, OR women married more than once** Write at (i) the date of first or only marriage Write at (ii) the date when that marriage ended *See Note 6*	**Country of birth** If born in Great Britain write "England", "Scotland", or "Wales", whichever applies *See Note 7.* If born in Ireland write "Northern Ireland" or "Irish Republic". If born elsewhere give the country of birth, e g Trinidad, Poland, or write "At Sea".	**For persons NOT born in Great Britain or Northern Ireland** *See Note 8* (a) If a citizen of the Commonwealth state at (i) citizenship, e g United Kingdom and Colonies, Indian, Canadian (b) If a citizen of the U K and Colonies state at (ii) whether citizen by birth, descent, naturalisation, registration, marriage, etc (c) For other persons state at (i) nationality, e g Italian, Polish, Yugoslav.
F (i) (ii)	G (i) (ii)	H (i) (ii)	J	K (i) (ii)
(i) (ii)	(i) (ii)	(i) (ii)		(i) (ii)

DIAGRAM 1

Questionnaire

Here is an example. Most people who enjoy looking at television can choose to look at either the BBC or ITV programme. Let us suppose that we wish to take a census of the preferences of the pupils in our school. A suitable form for this would be as follows.

Please complete the attached form to show (a) your name, (b) your year group, (c) your preference in television programmes, i.e. whether BBC or ITV.

Name		
Year Group (i.e. 1st yr., 2nd yr., etc.)		
I prefer	BBC	
	ITV	

Please indicate your preference with X.

DIAGRAM 2 (showing the results obtained after 100 pupils in each year had completed the questionnaire)

Year Group	1st year (Age 11–12)	2nd year (12–13)	3rd year (13–14)	4th year (14–15)	5th year (15–16)
Prefer BBC	36	33	38	29	39
Prefer ITV	64	67	62	71	61

These figures could be plotted on a graph (opposite) which will show us at a glance the relative popularity of the two programmes amongst our schoolfellows.

Now try to design suitable forms for the collection of the required information yourselves.

Exercise 10b

1. Design suitable forms to enable you to collect the heights and weights of the pupils in your form. See example (2) on page 86.

2. Ask your mathematics teacher to obtain for you the number of bottles of milk which are drunk by each year group in each month of the year. Design a suitable form to show these figures. Draw four (or five) graphs to show how the consumption of milk varies in each year every month. Can you draw any conclusions by looking at the differences of consumption in December and June?

3. The television companies are very interested to find out which

are the most popular programmes. Design a form which you could circulate amongst your friends with five of your favourite programmes noted on it and requiring your friends to place these programmes in order of preference.

4. Which is your favourite sport? How does your choice compare with that of your friends? Design a suitable form to find out this information and see if you can find out from this information the relative popularity of various sports among at least 100 people.

5. Use an atlas or other suitable book of reference (an Automobile Association Handbook is excellent) to estimate the distances of the following towns or cities from your home town: London, Birmingham, Manchester, Bolton, Liverpool, Norwich, Bristol, Exeter, Truro, Cardiff. Draw up a suitable table showing these distances. Exhibit these figures on a bar graph which will show you at a glance the relative distances of the towns and cities from your home town.

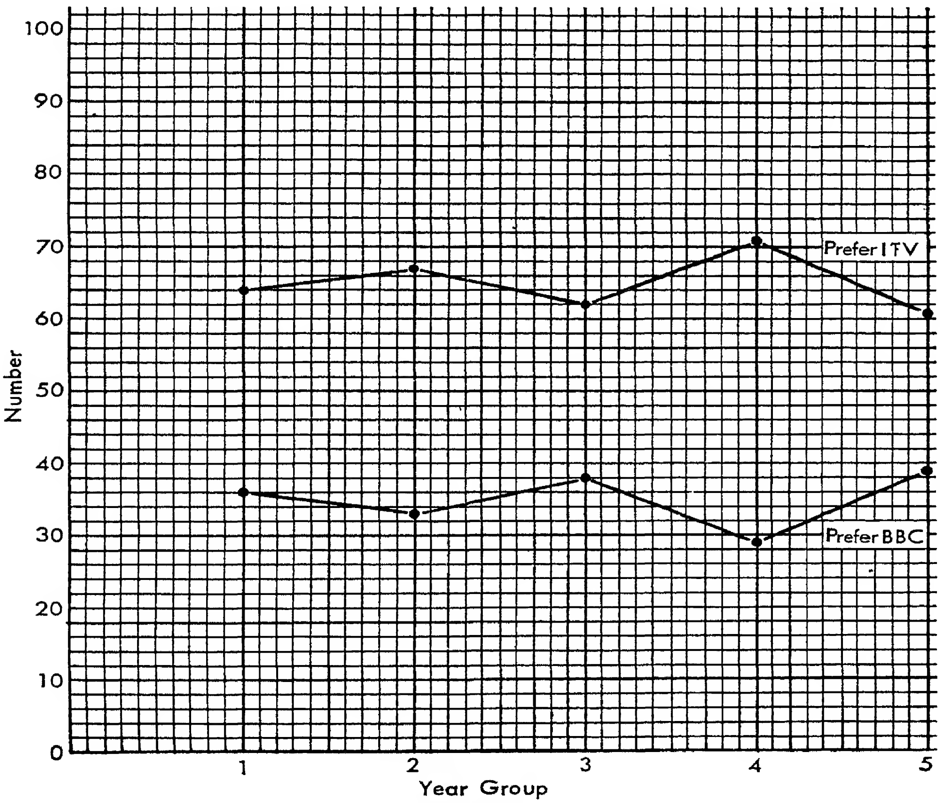


fig. 10.5

11 Area

AREA is the amount of surface covered.

Find the areas of the two figures in fig. (1) by counting the squares. How can you save counting all the squares individually in the second case?

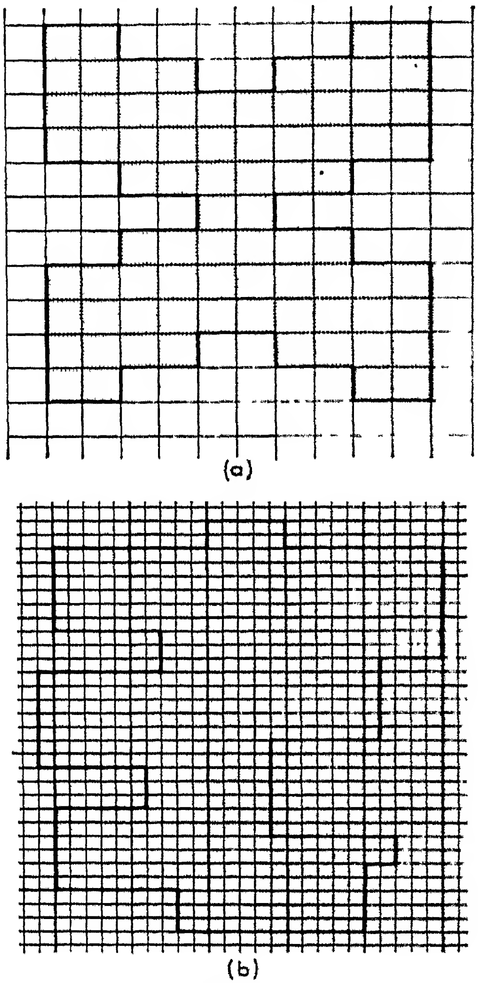


fig. 11.1

The areas of most shapes are not so easy to calculate as those in fig. 11.1. Look at the island shown in fig. 11.2. You will see that there are many squares which are cut by the coastline of the island. In

order to count the squares we include parts of a square which are greater than a half, counting them as a whole square; and we leave out any parts of squares which are less than a half. Find the area of the island.

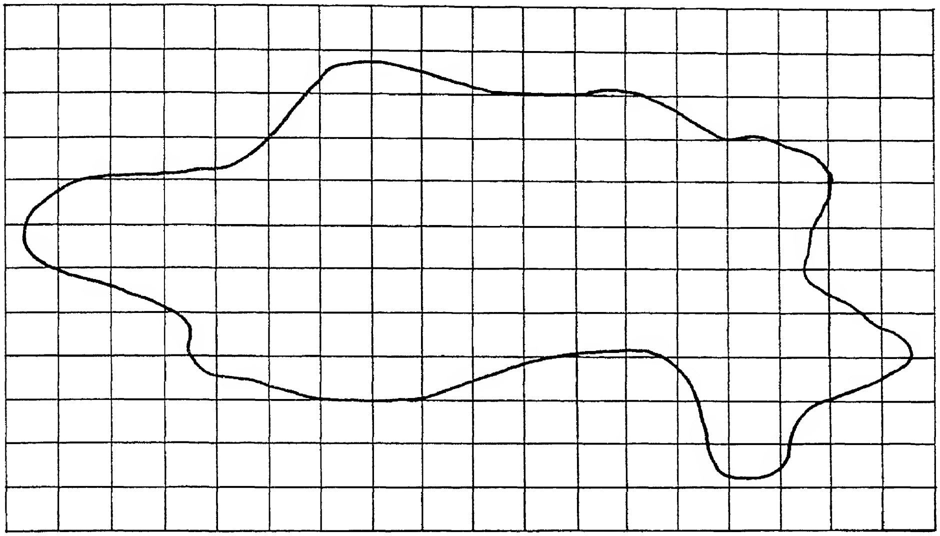


fig. 11.2

Now find the area of the island in fig. 11.3. With this particular area we do not have to count every square, for we can use the same helpful method as in fig. 11.1(b).

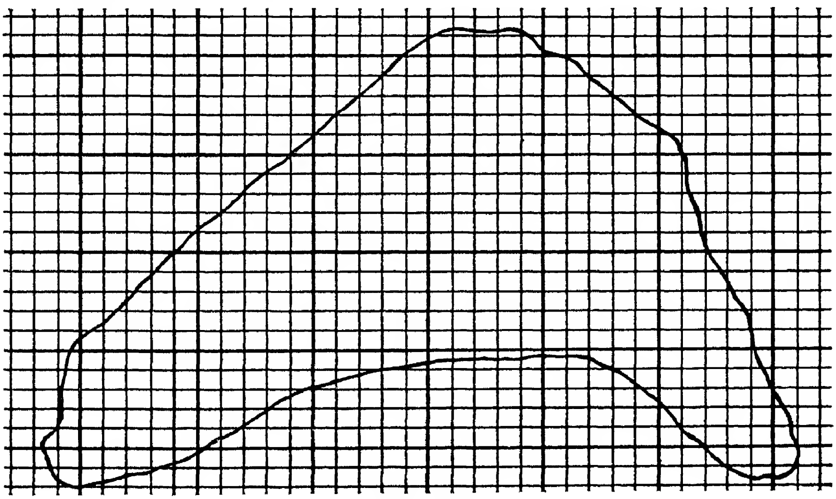


fig. 11.3

If you have worked accurately you should have found that there are over four times as many squares in the area of the second island than in the area of the first island. This does not mean however that the size of the second island is over four times that of the first island, for if you look at the two figures you notice that the sizes of the squares covering the islands are different.

In order to compare the actual sizes of the two islands you must use the same size squares for your unit of measure. In fig. 11.4 the first island is now covered by squares which are the same size as those in fig. 11.3. Find the area by counting the squares and see which island is the largest.

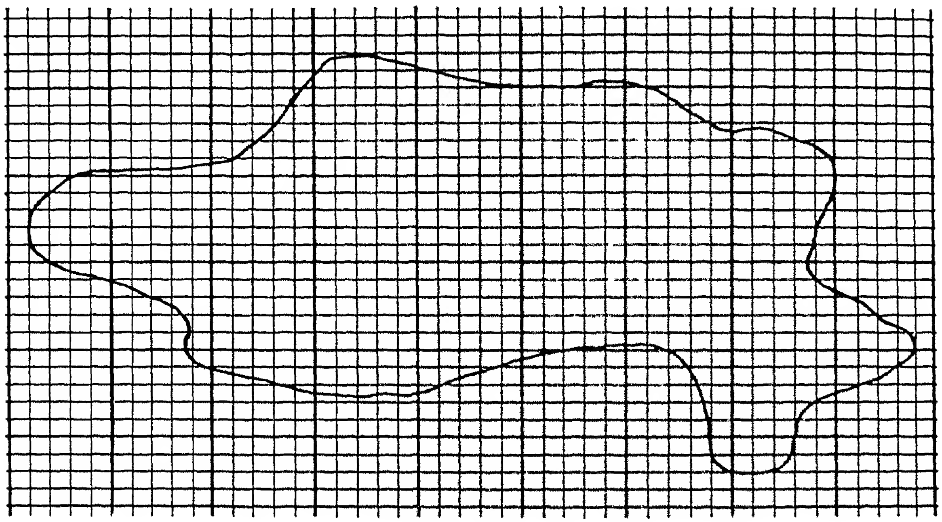
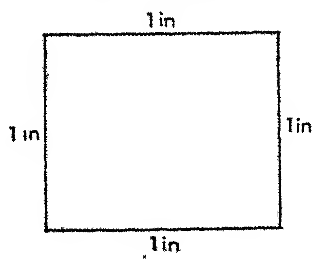


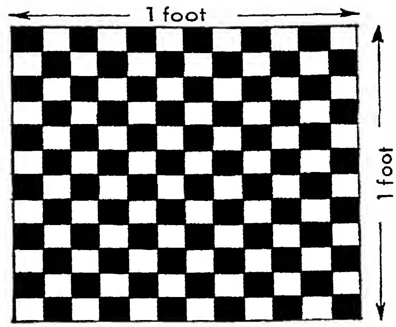
fig. 11.4

To compare areas of different shapes you must use the same size squares. It has become the custom to use squares with a side of length 'one'. For instance, if the area is small you use a square with a side one inch; if the area is larger you can use a square with side one foot; if the area is larger still, you could use a square with side one yard, and so on. A square with side one inch is said to have an area of one square inch.



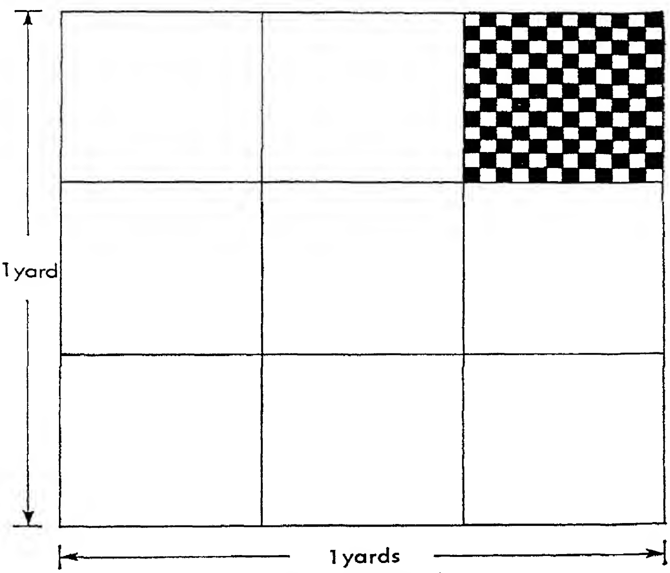
Area = 1 sq. inch.
fig. 11.5

A square with side one foot is said to have an area of one square foot.



Area = 1 sq foot.
fig. 11.6

A square with side one yard is said to have an area of one square yard.



Area = 1 sq yard
fig. 11.7

A square with side one mile is said to have an area of one square mile. The square mile would be the unit of measure used when finding the area of large masses of land such as the islands talked about earlier.

Important Questions

How many square inches in 1 square foot?
How many square feet in 1 square yard?

Exercise 11a

1. The area of a lawn is 5,148 sq. ft. How many sq. yd. is this?

2. How many square inches in **(a)** $\frac{1}{2}$ sq. ft. **(b)** $\frac{1}{4}$ sq. ft. **(c)** $\frac{1}{3}$ sq. ft. **(d)** $1\frac{1}{2}$ sq. ft. **(e)** 4 sq. ft.?

3. How many square feet in **(a)** 288 sq. in. **(b)** 576 sq. in. **(c)** 1,296 sq. in. **(d)** 648 sq. in.?

4. How many square feet in **(a)** 4 sq. yd. **(b)** 9 sq. yd. **(c)** $\frac{1}{3}$ sq. yd. **(d)** $\frac{2}{3}$ sq. yd. **(e)** $\frac{1}{2}$ sq. yd.?

5. What part of a square foot is **(a)** 36 sq. in. **(b)** 72 sq. in. **(c)** 48 sq. in. **(d)** 108 sq. in.

It is easy to find the areas of rectangles by dividing them into squares. For example a rectangle 6 in. × 2 in. has an area of 12 square inches, as shown in fig. 11.8.

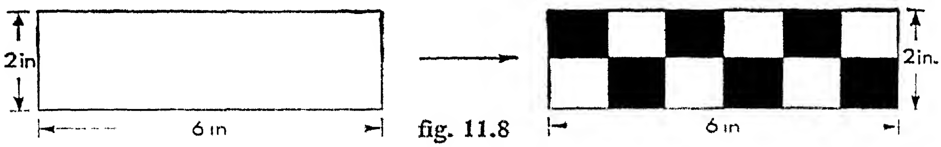


fig. 11.8

Fig 11.9 shows that a rectangle 3 in. × 5 in. has an area of 15 square inches.

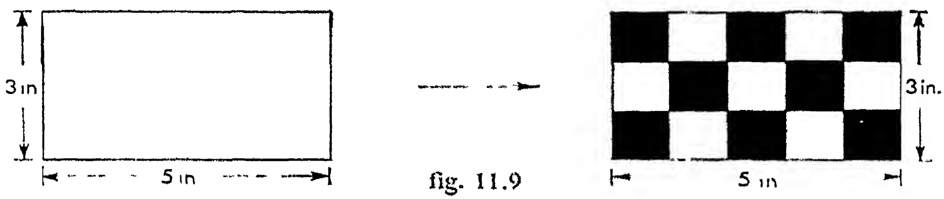


fig. 11.9

From the above examples you should see that by calculating the number of rows of squares, and calculating the number of squares in each row there is a simple way of finding the areas of rectangles. This method gives us a rule for rectangles.

It is: AREA OF RECTANGLE = LENGTH × BREADTH.

This rule must be learnt, but it applies ONLY TO RECTANGLES.

The above rule applies even when there is not a complete row of squares. Fig. 11.10 shows an example of this.

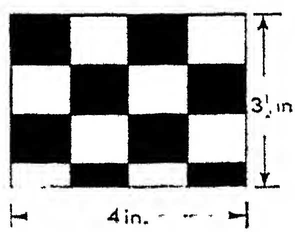


fig. 11.10

We have a rectangle 4 in. × $3\frac{1}{2}$ in., which gives 4 rows, and in each row there are $3\frac{1}{2}$ squares; thus the area is $4 \times 3\frac{1}{2} = 14$ sq. in. The following exercise deals with areas of rectangles, so you will be using the rule that you have just learnt.

Exercise 11b

1. Fill in this table:

LENGTH OF RECTANGLE	BREADTH OF RECTANGLE	AREA OF RECTANGLE
6 in.	2 in.	
8 in.	3 in.	
6 in.	$2\frac{1}{2}$ in.	
7 in.		28 sq. in.
5 ft.		30 sq. ft.
	8 yd.	48 sq. yd.
	9 ft.	81 sq. ft.
6 ft.		4 sq. yd.
	24 in.	4 sq. ft.
	6 in.	4 sq. ft.

2. By supposing the figure divided into two rectangles as shown by the dotted line, find the area of the L-shaped figure:

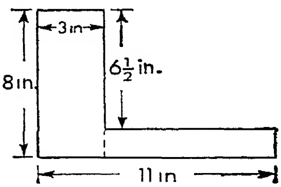


fig. 11.11

3. By supposing the figure divided into 3 rectangles, find the area of the cross section of the girder shown here:

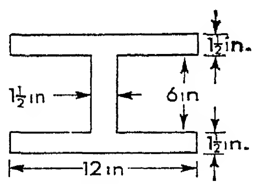


fig 11.12

4. Find the areas of these letters:

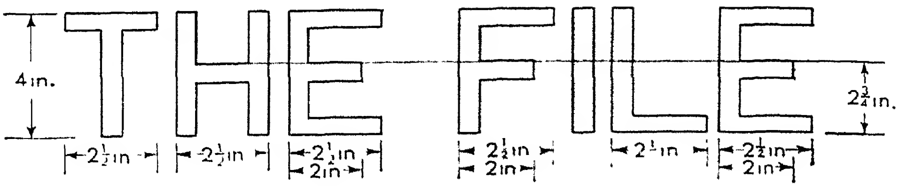


fig. 11.13

Consider the square in fig. 11.14. Suppose each small square is one inch long and one inch wide, then the total area of the large square is 144 square inches or one square foot. If you cut along the curved line the square could be rearranged to give the shape shown in fig. 11.15. Since the surface covered is still the same, the area of the new shape must be 144 square inches or one square foot. This is a very important point for you to learn; a shape does not have to be a square to have an area of one square foot (or a square inch, etc.), and we cannot find all areas by multiplying 'length × breadth' as we did with the rectangles.

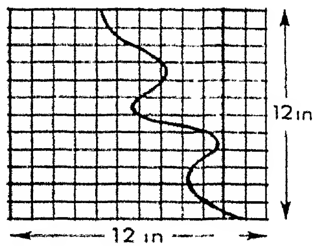


fig. 11.14

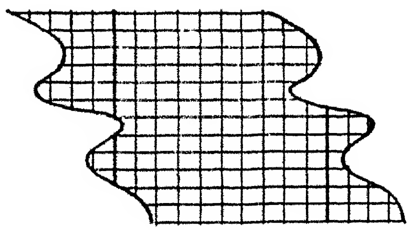


fig. 11.15

Exercise 11c

Assume each small square in the following exercise is 1 in. × 1 in.; find in square feet and square inches the areas of the given shapes. In this exercise we must use the same trick as when we found the area of the islands. That is we count as complete squares all part squares larger than a half, and ignore all parts smaller than a half. Can you see any quick methods to save some work?

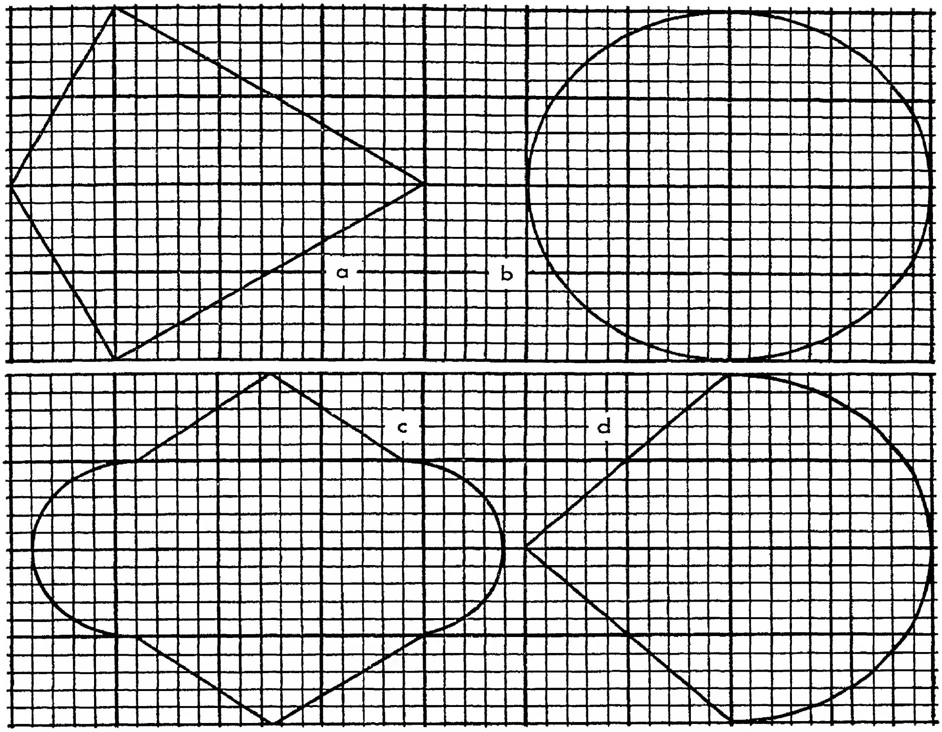


fig. 11.16

Exercise 11d

1. The edges of a cuboid are 6 in.–8 in.–10 in. What is its total surface area?

2. The living-room measured 16 ft. × 12 ft. while the dining-room measured 18 ft. × 11 ft. Which was the larger room?

3. How many square yards in a lawn 30 ft. by 21 ft.? How much would it cost to turf it at 3*s.* 6*d.* per sq. yd.?

4. How much would it cost to seed the above lawn if seed is 8*s.* per lb. and we sow at the rate of 2 oz. per sq. yd.?

5. Find the cost of a carpet 12 ft. × 15 ft. at 24*s.* per sq. yd.

6. Find the cost of plywood 3 ft. 6 in. × 4 ft. at 3*s.* per sq. ft.

7. A rectangle has an area of 30 sq. in. If its width is 4 in., what is its length?

8. How many pieces of paper 4 in. square can be cut from a piece 2 ft. 6 in. × 4 in. wide? What area is left over?

9. A piece of paper 6 in. × $4\frac{1}{2}$ in. is ruled in squares of $\frac{1}{4}$ in. × $\frac{1}{4}$ in. How many would there be?

10. Squared paper is ruled into $\frac{1}{10}$ in. squares. How many are there on a piece 10 in. × 5 in.?

11. What will it cost to cover a floor 14 ft. × 11 ft. 6 in. with floor covering at 2*s*. 9*d*. per sq. yd.?

12. How many wood blocks 6 in. × 2 in. will be needed to cover a floor 12 ft. long × 10 ft. 6 in. wide?

When talking about wood, hardboard, etc., the size required is given by the lengths of the sides. A piece of wood would be said to be 'four feet by two feet'.
If a piece of wood is square, instead of saying 'four feet by four feet' it is said to be 'four feet square'.
Thus to cut a piece of wood to four feet square is not the same as to cut it to four square feet. See fig. 11.18.

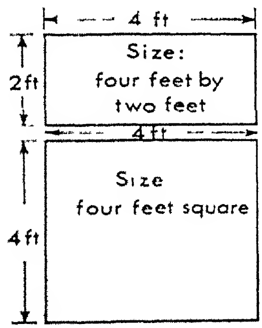


fig. 11.17

What is the difference in area between four square feet and four feet square?
What is the difference, if any, between two pieces of wood, one being cut three feet square and the other to three square feet?

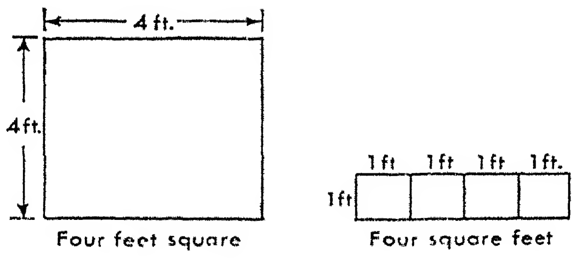


fig. 11.18

Land Measures

Land is, of course, always measured by area. In history you can read of the land being divided into sections for the villagers. These sections

were usually one furrow long (1 furlong) and 'a chain' wide. When measurements became standardized the chain was set at 22 yards and the furlong at 10 chains (220 yards). This gave the important land measure of 1 acre.

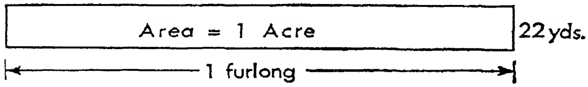


fig 11.19

1 acre = 4,840 sq. yd.
This is the measure used today.

Exercise 11e

1. Find in acres the area of a field 121 yd. × 160 yd.
2. How many plants will be in a field of $2\frac{1}{4}$ acres at the rate of 4 per sq. yd.?
3. How many square yards in a square mile?
4. How many acres in a square mile?
5. How large, in acres, is a playing field which contains 6 football pitches 110 yd. × 80 yd. each if a total of 5,000 sq. yd. is allowed around the pitches for spectators?

Exercise 11f

PROBLEMS ON BORDERS

1. A carpet 12 ft. × 10 ft. is laid in a room 14 ft. square. How much of the floor is left uncovered?
2. A photograph 18 in. × 15 in. is mounted on a card 2 ft. × $1\frac{1}{2}$ ft. What area of card is uncovered?
3. Find the shaded area in the diagrams.

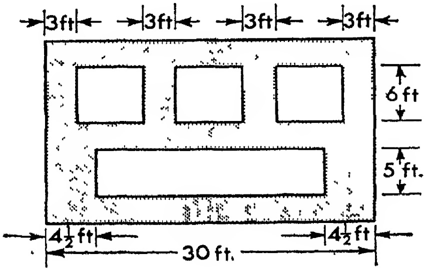


fig. 11.20

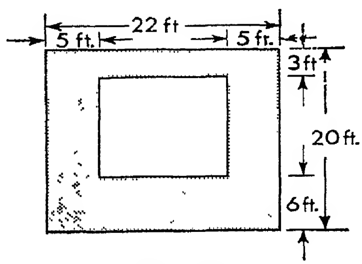


fig. 11.21

4. A lawn 30 yd. × 18 yd. is surrounded by a 2 ft. 6 in. path. Find the area of the path.

5. The diagram is of a garden with two flower-plots each 20 ft. × 12 ft. The grass surround is everywhere 5 ft. wide. What is the area of the whole garden? What is the area of flowers? What is the area of grass?

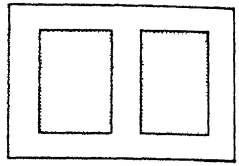


fig 11.22

Square Numbers

Count the number of squares in each of the following six figures. They are the square numbers; it will help you if you learn them. What are the next four square numbers?

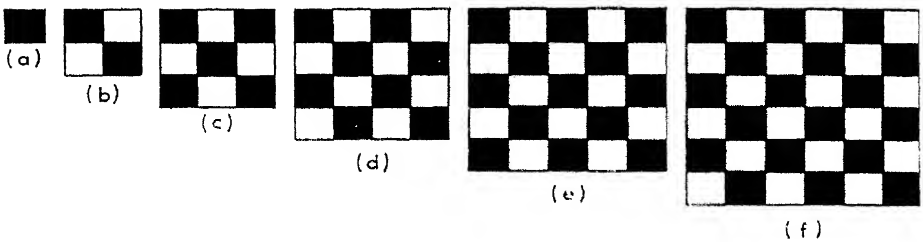


fig. 11.23

Note these numbers can be formed in this way:

(a) $= 1$
(b) $= 1 + 3$
(c) $= 1 + 3 + 5$
(d) $= 1 + 3 + 5 + 7$
(e) $= 1 + 3 + 5 + 7 + 9$
(f) $= 1 + 3 + 5 + 7 + 9 + 11$

or in this way:

(a) 1
(b) $1 + 2 + 1$
(c) $1 + 2 + 3 + 2 + 1$
(d) $1 + 2 + 3 + 4 + 3 + 2 + 1$
(e) $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1$
(f) $1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1$

Can you see how these numbers have been built up by taking different arrangements of the small squares?

12 Volume

A solid object occupies space. The amount of space it occupies is called its VOLUME. The larger the object, the larger its volume. If an object is a vessel, like a cup, a milk-bottle, or a jug, its volume, or capacity, is the amount of space which can be filled.

When you were dealing with areas, the surface was divided into squares and the area was found by counting the squares. This gave the area in square inches, or square feet, etc. These were the 'units' for measuring area.

In order to compare the volumes of different solids it is again essential to have some unit of measure. The unit which has been chosen is the cube. All volumes are compared to the volume of a cube, and we cut up the volume into cubes. Just as squares with side one were used for areas so cubes with side one are used for volumes.

A cube with side 1 inch is said to have a volume of 1 cubic inch.

fig. 12.1

A cube with side 1 foot is said to have a volume of 1 cubic foot.

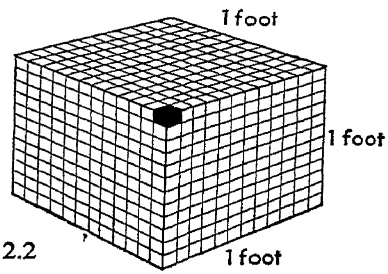


fig. 12.2

A cube with side 1 yard is said to have a volume of 1 cubic yard.

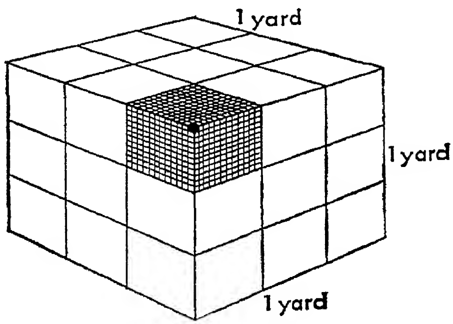


fig. 12.3

How many cubic inches in a cubic foot?

How many cubic feet in a cubic yard?

How many cubic yards in a cubic mile?

If you had a plasticine ball that could be reshaped into 4 cubes of side one inch you would say its volume was 4 cubic inches.

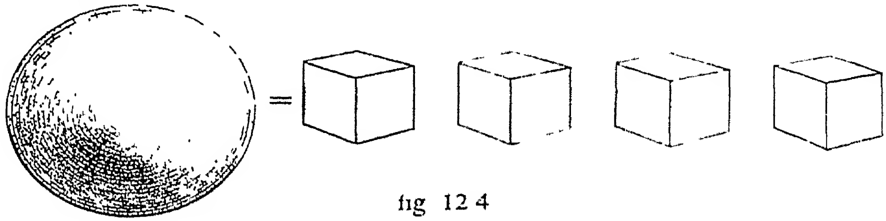


fig 12 4

And if you had a pyramid that could be reshaped into 2 cubes of side one inch you would say its volume was 2 cubic inches.

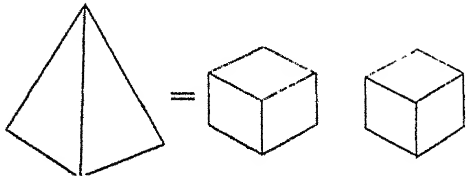


fig. 12.5

It is then easy to see how much larger the ball is than the pyramid. It is often possible to find the volume of a solid by simply dividing it into cubes and counting the cubes, as in the diagram:

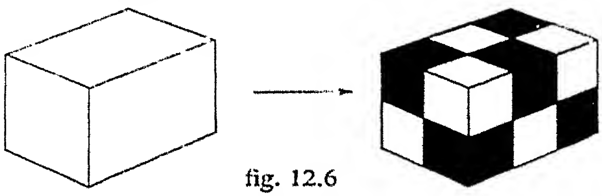


fig. 12.6

Here you see that a block 3 in. × 2 in. × 2 in. has a volume of 12 cubic inches. Thus an easy way of finding the number of cubic inches would be to find the number of layers of cubes and the number of cubes in each layer. The volume is found by multiplying the number of layers by the number of cubes in each layer.

Exercise 12a

Suppose you have a number of inch cubes, how many would you need to make blocks of the following sizes?
Draw simple sketches to help you.

e.g. Block length 4 in., breadth 3 in., height 3 in.
No. in each layer $4 \times 3 = 12$
No. of layers 3
$\therefore$ No. of inch cubes required $3 \times 12 = 36$

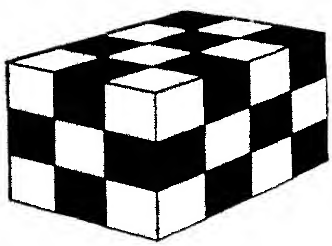


fig. 12.7

1.	Length	6 in.	breadth	3 in.	height	1 in.
2.	,,	6 in.	,,	3 in.	,,	2 in.
3.	,,	6 in.	,,	3 in.	,,	5 in.
4.	,,	8 in.	,,	4 in.	,,	1 in.
5.	,,	8 in.	,,	4 in.	,,	6 in.
6.	,,	7 in.	,,	3 in.	,,	4 in.
7.	,,	7 in.	,,	5 in.	,,	2 in.
8.	,,	7 in.	,,	5 in.	,,	4 in.
9.	,,	14 in.	,,	5 in.	,,	4 in.
10.	,,	1 ft. 6 in.	,,	$\frac{1}{2}$ ft.	,,	5 in.

Can you see how to do questions **8** and **9** quickly after doing question **7**?

Having worked out the problems above, you should see the rule for working out the volumes. It is simply

LENGTH IN INCHES × BREADTH IN INCHES × HEIGHT IN INCHES = VOLUME IN CUBIC INCHES.

Exercise 12b

1. If you have a large block of 30 cubic inches and its length is 3 in. and breadth 2 in., how high is it?

2. Complete this table:

	LENGTH	BREADTH	HEIGHT	VOLUME
(a)	15 in.	11 in.	4 in.	—
(b)	$6\frac{1}{2}$ in.	4 in.	3 in.	—
(c)	$8\frac{1}{2}$ in.	$5\frac{1}{2}$ in.	2 in.	—
(d)	10 in.	8 in.	—	400 cu. in.
(e)	11 ft.	—	7 ft.	693 cu. ft.
(f)	$10\frac{1}{2}$ yd.	—	8 yd.	420 cu. yd.
(g)	—	8 ft.	6 ft.	504 cu. ft.
(h)	1 ft. 4 in.	1 ft.	3 in.	—
(i)	1 ft. 5 in.	7 in.	4 in.	—
(j)	—	18 in.	8 in.	$1\frac{1}{2}$ cu. ft.

Measuring Irregular Solids

The volume of the majority of objects cannot be found by measuring the *length, breadth and height* because these are not regular. Another method must be used. Here is an experiment to help you measure the volume of odd shapes such as screws, nuts, pieces of coal, stones, etc. Make an inch cube without a top and cover the inside with a thin layer of paraffin wax (use thick waxed paper if you can get some). Ask your mother for a jar—a narrow one is best. Stick on the outside a piece of paper for a scale. Fill your cube with water and pour it into the jar, marking the paper level with the top of the water. Continue this until you have filled the jar about three-quarters full. Count from the bottom and number your marks in fives, each mark showing a cubic inch of water.

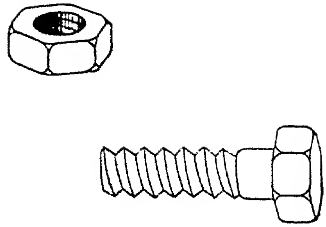


fig. 12.8

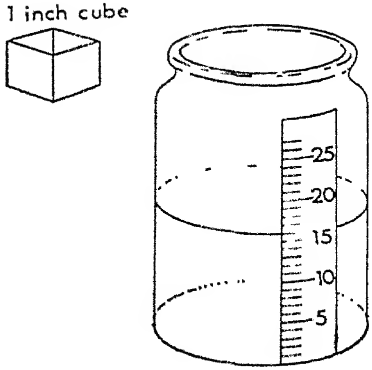


fig. 12.9

How to Use the Measuring Jar

Half fill your jar and note how much water is in it. Put in the odd shape you wish to measure. Take a fresh reading of the water level. The increase will be equal to the volume of the object.

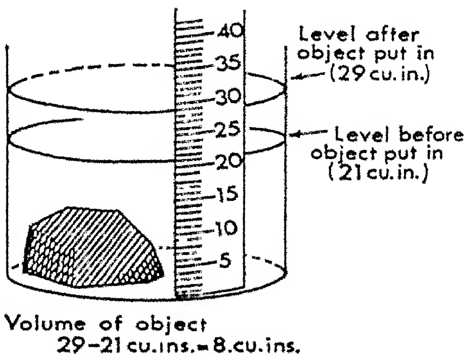


fig. 12.10

(1) Use your jar to find the volumes of the following odd-shaped articles: (*a*) a bolt (*b*) a nut (*c*) a piece of coal (*d*) a table-tennis ball.

(2) Use your jar to find the volume that an egg cup will hold; a tea-cup; an empty ink-bottle.

The rule that you learnt for finding volumes applies only to cuboids (rectangular prisms) and is always true, as long as we keep our lengths in the same 'units', i.e. they must all be in inches, or they must all be in feet, or all in yards.

Exercise 12c

1. Find the volume of a brick 9 in. $\times$ $4\frac{1}{2}$ in. $\times$ 3 in.

2. If a $\frac{1}{2}$ lb. of margarine measures $4\frac{1}{2}$ in. $\times$ $2\frac{3}{4}$ in. $\times$ $1\frac{1}{2}$ in., how many cubic inches does 1 lb. of margarine occupy?

3. How many cubic inches are there in this casting?
Do this two ways and check your answers.
(a) Imagine it solid, then take away the piece missing.
(b) Imagine it in 3 sections.

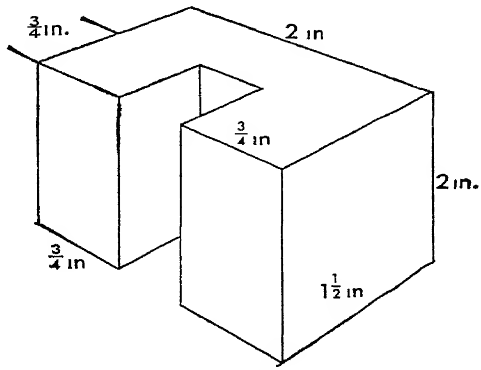


fig. 12.11

4. Find the volume of a plank of wood 8 in. $\times$ 3 in. and 12 ft. long.

5. How many cubic feet of concrete to lay a 25 ft. path, 3 ft. wide and 4 in. thick?

6. What volume of concrete is needed to make a hollow post, as in the diagram, 20 ft. high?

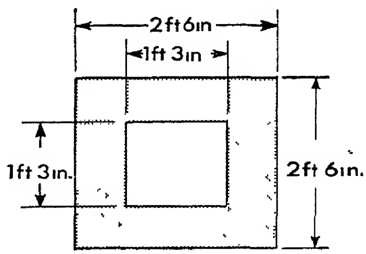


fig. 12.12

7. Find the volume of wood used to make the following open boxes:

(**a**) 10 in. × 8 in. × 9 in.; the material 2 in. thick

(**b**) 9 in. × 6 in. × 8 in.; the material 1 in. thick

(dimensions given are external)

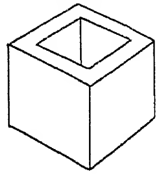


fig. 12.13

8. What volume of wood is needed to make the following *closed* boxes:

(**a**) 14 in. × 10 in. × 9 in., the material 1 in. thick

(**b**) $1\frac{1}{2}$ ft. × $\frac{3}{4}$ ft. × $\frac{1}{2}$ ft., the material $\frac{1}{2}$ in. thick

9. Find the volume of these solids:

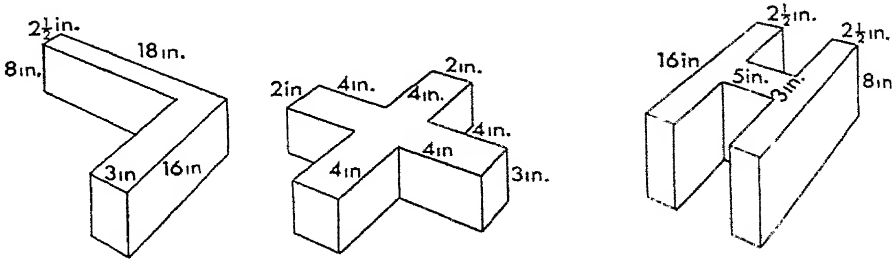


fig. 12.14

10. A swimming bath is 100 ft. long 40 ft. wide and has a depth of 6 ft. What volume of water does it hold?

11. How many gallons are there if 1 cu. ft. holds $6\frac{1}{4}$ gallons?

12. What is the weight of the water if one pint of water weighs a pound and a quarter?

13 Everyday Calculations with Compound Units

The length of a dining room is 12 ft. $3\frac{1}{2}$ in.
The cost of the groceries is £3 14*s*. 2*d*.
The weight of the lorry is 2 tons 3 cwt. 1 qtr. 10 lb.
The time for the journey was 7 hr. 35 min.
These are everyday quantities and they all require more than one unit to express them. When more than one unit is used, e.g. feet and inches, the quantity is said to be expressed in 'compound units'.

Tables to be Learnt

(1) *Money*	12 pence	=	1 shilling
	20 shillings	=	1 pound (£1)
	240 pence	=	1 pound
(2) *Length*	12 inches	=	1 foot
	3 feet	=	1 yard
	36 inches	=	1 yard
	1760 yards	=	1 mile
	5280 feet	=	1 mile
	8 furlongs	=	1 mile
(3) *Weight*	16 ounces	=	1 pound
	14 pounds	=	1 stone
	2 stones	=	1 quarter
	4 quarters	=	1 hundredweight
	20 hundred-weights	=	1 ton
	28 pounds	=	1 quarter
	112 pounds	=	1 hundredweight
	2240 pounds	=	1 ton
(4) *Time*	60 seconds	=	1 minute
	60 minutes	=	1 hour
	24 hours	=	1 day
(5) *Capacity*	2 pints	=	1 quart
	4 quarts	=	1 gallon
	8 pints	=	1 gallon

Note. The history of these tables is very interesting. Find a book in the library which tells about them.
Conversion—changing a quantity in compound units into its equivalent in the smallest units or vice versa.
e.g. Change £3 14*s.* 7*d.* to pence.

NOTES

Start with the biggest unit, in this case £'s.
Change to shillings by multiplying by 20.
Add these shillings to the original number in the next column and multiply the total by 12 to change to pence.
Add this number of pence to the original number to give the answer.

£	*s.*	*d.*
3 ×	14	7
20	60	888
60	74 ×	895
	12	
	888	

∴ £3 14*s.* 7*d.* = 895*d.*

This process is the same whatever the units are.

e.g. Change 3 cwt. 3 qtr. 12 lb. to pounds.

Here multiply cwts. by 4 to change them to quarters. Then multiply quarters by 28 to change them to pounds.

cwt.	qtr.	lb.
3 ×	3	12
4	12	420
12	15 ×	432
	28	
	300	
	120	
	420	

∴ 3 cwt. 3 qtr. 12 lb. = 432 lb.

The reverse process uses division

e.g. Change 683*d.* to £ *s.* *d.*

12	683*d.*	
$2\cancel{0}$	$5\cancel{6}$*s.* rem. 11*d.*	
	£2 16*s.*	11*d.*

∴ 683*d.* = £2 16*s.* 11*d.*

Divide the pence by 12 to give shillings.
The remainder is in pence and is the first part of the answer.
Divide the shillings by 20 (cross off the 0 of the 20 and the 6 of the 56.

This 6 is written down as part of the remainder. Divide by 2. If there is a remainder of 1, as here, write it in front of the 6 giving 16*s*.).

Another example. Express 175 inches in yards, feet and inches.

$$\begin{array}{r|lll} 12 & 175 \text{ in.} & & \\ \hline 3 & 14 \text{ ft.} & 7 \text{ in.} & \\ \hline & 4 \text{ yd.} & 2 \text{ ft.} & 7 \text{ in.} \end{array}$$

$$\therefore 175 \text{ in.} = 4 \text{ yd. } 2 \text{ ft. } 7 \text{ in.}$$

Divide inches by 12 to give feet, the remainder 7 is in inches.
Divide the feet by 3 to give yards, the remainder is in ft.

Exercise 13a

1. Change into pence.

(i)	14*s*. 7*d*.	**(vi)**	£2 0*s*. 10*d*.
(ii)	£1 7*s*. 2*d*.	**(vii)**	£7 19*s*. 1*d*.
(iii)	£3 17*s*. 4*d*.	**(viii)**	£14 3*s*. 0*d*.
(iv)	£2 14*s*. 11*d*.	**(ix)**	£17 13*s*. 8*d*.
(v)	£6 4*s*. 3*d*.	**(x)**	£21 15*s*. 2*d*.

2. Change into inches.

(i)	2 ft. 8 in.	**(vi)**	8 yd.
(ii)	1 yd. 2 ft. 3 in.	**(vii)**	7 yd. 2 ft. 11 in.
(iii)	3 yd. 1 ft. 4 in.	**(viii)**	6 yd. 0 ft. 2 in.
(iv)	2 yd. 0 ft. 11 in.	**(ix)**	17 ft. 11 in.
(v)	4 yd. 2 ft. 7 in.	**(x)**	14 yd. 2 ft. 10 in.

3. Change into quarters.

(i)	2 cwt. 3 qtr.	**(iv)**	3 tons 0 cwt. 3 qtr.
(ii)	17 cwt. 2 qtr.	**(v)**	5 tons 13 cwt. 2 qtr.
(iii)	2 tons 3 cwt. 1 qtr.		

4. Change into pounds (Americans express their body weight in pounds only).

(i)	1 st. 3 lb.	**(iv)**	10 st. 7 lb.
(ii)	5 st. 12 lb.	**(v)**	12 st. 13 lb.
(iii)	6 st. 11 lb.		

5. Change into ounces.

(i)	1 lb. 7 oz.	**(iv)**	6 lb. 15 oz.
(ii)	2 lb. 13 oz.	**(v)**	5 lb. 2 oz.
(iii)	3 lb. 5 oz.		

6. Change into seconds.

(i) 12 min. 5 sec.
(ii) 37 min. 14 sec.
(iii) 1 hr. 0 min. 23 sec.
(iv) 47 min. 57 sec.
(v) 55 min. 31 sec.

7. Change into £ *s.* *d.*

(i) 79*d.* (No pounds)
(ii) 215*d.* (,, ,,)
(iii) 100*d.* (,, ,,)
(iv) 487*d.*
(v) 335*d.*
(vi) 1073*d.*
(vii) 1872*d.*
(viii) 999*d.*
(ix) 731*d.*
(x) 1440*d.*

8. Change into yards, feet and inches.

(i) 35 in.
(ii) 47 in.
(iii) 59 in.
(iv) 100 in.
(v) 150 in.
(vi) 147 in.
(vii) 252 in.
(viii) 216 in.
(ix) 187 in.
(x) 325 in.

9. Change into pounds and ounces.

(i) 37 oz.
(ii) 92 oz.
(iii) 105 oz.
(iv) 192 oz.
(v) 200 oz.

10. Change into hours and minutes.

(i) 67 min.
(ii) 145 min.
(iii) 200 min.
(iv) 173 min.
(v) 402 min.

The Four Rules for Compound Units

(1) ADDITION

EXAMPLE

NOTE

Add all the pence and write the total underneath. Divide by 12 to change them to shillings. Put the shillings in the next column and the remainder in the pence column. Then add all the shillings and divide them by 20.

Add	£	*s.*	*d.*
	4	2	8
	1	17	7
		14	3
	6	14	6
	1	1	12 | 18*d.*
	5	33	1*s.* 6*d.*
	6	20 | 34	
		£1 14*s.*	

The working underneath the answer line is only an aid to help you if you are not sure of the method. As soon as possible all working should be done mentally.

(2) SUBTRACTION

EXAMPLE

Subtract

yd.	ft.	in.
3	1	4
1	2	11
1	1	5

METHOD

11 cannot be taken from 4. Add 12 in. (1 ft.) to the 4 making 16. Then 16 — 11 = 5 in. Balance up by adding 1 ft. to the 2 ft. making 3. Again 3 cannot be taken from 1. Add 3 ft. (1 yd.) to the 1 making 4. Then 4 — 3 = 1 ft. Balance up by adding 1 yd. to the 1 yd. in the bottom line making 2. Then 3 — 2 = 1 yd. giving the answer 1 yd. 1 ft. 5 in.

NOTE

Do not use carrying figures unless you have to.

Exercise 13b

Add

1.

£	s.	d.	
2	3	11	+
1	4	7	

2.

£	s.	d.	
2	14	3	+
3	7	5	

3.

£	s.	d.	
5	17	8	+
3	4	9	

4.

£	s.	d.	
3	15	10	+
4	17	8	

5.

£	s.	d.	
1	3	6	+
4	7	11	
5	15	2	

6.

£	s.	d.	
3	0	4	+
2	16	9	
14	7	10	

7.

£	s.	d.	
14	7	10	+
28	19	4½	
6	7	11½	

8.

£	s.	d.	
3	7	0	+
14	19	2	
	18	6½	
13	5	9	

9. £3 10*s.* 4½*d.* + £13 0*s.* 5*d.* + £11 + £19 19*s.* 7*d.*

10. £13 14*s.* 10*d.* + £4 19*s.* 5*d.* + £24 13*s.* 11*d.* + £36 11*s.* 10*d.* + £18 0*s.* 4*d.*

11.

lb.	oz.
7	4 +
3	14

12.

lb.	oz.
3	6 +
2	14
5	10

13.

lb.	oz.
4	8 +
3	0
7	15
8	3

14.

tons	cwt.
14	7 +
3	19

15.

tons	cwt.
3	17 +
4	9
5	12

16.

tons	cwt.	qtr.
3	4	2 +
1	17	3

17.

tons	cwt.	qtr.
14	16	1 +
3	12	3
5	17	2

18.

tons	cwt.	qtr.
3	12	3 +
4	6	2
14	5	3

19.

st.	lb.
4	12 +
6	18

20.

st.	lb.
13	10 +
6	8
9	12

21.

ft.	in.
2	10 +
1	8
5	9

22.

ft.	in.
3	6 +
4	8
	9

23.

yd.	ft.	in.
1	2	5 +
3	2	1
4	1	9

24.

yd.	ft.	in.
6	1	7 +
3	0	9
	2	11

25.

yd.	ft.	in.
7	2	4 +
8	2	6
3	1	2

26. 2 ml. 3 fur. 70 yd. + 3 ml. 7 fur. 112 yd. + 1 ml. 4 fur. 193 yd.

27. 14 tons 13 cwt. 3 qtr. + 3 tons 2 qtr. + 17 cwt. 1 qtr. + 10 tons 3 cwt. 1 qtr.

28. 4 hr. 31 min. 25 sec. + 2 hr. 47 min. 36 sec. + 3 hr. 55 min. 18 sec.

Exercise 13c

Subtract

1.

s.	*d.*	
3	3	—
1	10	

2.

s.	*d.*	
16	4	—
10	11	

3.

£	*s.*	*d.*	
6	3	9	—
4	18	4	

4.

£	*s.*	*d.*	
4	0	0	—
2	18	5	

5.

£	*s.*	*d.*	
5	3	0	—
1	13	8	

6.

£	*s.*	*d.*	
7	14	6	—
3	18	9	

7.

£	*s.*	*d.*	
12	13	10	—
3	14	9	

8.

£	*s.*	*d.*	
6	0	5	—
3	14	10	

9.

£	*s.*	*d.*	
3	7	0	—
1	0	9	

10. Take £1 14*s.* 2*d.* from £3 2*s.* 1*d.*

11. What is the difference between £6 18*s.* 7*d.* and £9 3*s.* 4*d.*?

12.

lb.	oz.	
3	14	—
1	7	

13.

lb.	oz.	
9	8	—
3	14	

14.

lb.	oz.	
7	0	—
3	11	

15.

tons	cwt.	
5	17	—
2	19	

16.

tons	cwt.	
13	0	—
3	15	

17.

cwt.	qtr.	
7	1	—
3	3	

18.

tons	cwt.	qtr.	
3	14	2	—
1	17	3	

19.

tons	cwt.	qtr.	
4	0	3	—
2	11	1	

20.

tons	cwt.	qtr.	
5	1	0	—
3	17	3	

21.

st.	lb.	
6	7	—
3	10	

22.

st.	lb.	
12	3	—
8	9	

23.

st.	lb.	
7	0	—
3	11	

24.

ft.	in.
4	7 —
1	9

25.

ft.	in.
9	3 —
2	11

26.

yd.	ft.	in.
3	2	7 —
1	1	9

27.

yd.	ft.	in.
5	0	7 —
2	2	3

28.

yd.	ft.	in.
4	1	8 —
3	0	9

29.

ml.	fur.
8	3 —
4	7

30. What is the difference between 8*s.* 3*d.* and 11*s.* 2*d.*?

31. Take 4 ml. 3 fur. 100 yd. from 5 ml.

32. Subtract 7 yd. 2 ft. from 8 yd. 1 ft. 7 in.

(3) MULTIPLICATION

Here the working bears some resemblance to addition. After all multiplication is really continued addition.

e.g.

£		*s.*		*d.*	
3		16		7 ×	
				5	
19		2		11	
4		2	12	35	2*s.* 11*d.*
15		80			
19	$2\cancel{0}$	$8\cancel{2}$			
		£4 2*s.*			

NOTE

Multiply 7*d.* × 5 = 35 and write 35*d.* underneath the answer line. Change into *s.* and *d.* Write the pence in the answer and transfer the shillings to the *s.* column. Then multiply 16*s.* × 5 = 80*s.* and add this to the 2*s.* Change the total to £ and *s.* Write the shillings in the answer line and transfer the pounds to the £ column. Then multiply £3 × 5 = 15 and add all the pounds together, to complete the answer.

You will find it more difficult to do the necessary working mentally, but when you can, leave the working out.

Exercise 13d

Multiply

1. £2 3*s*. 8*d*. × 3
2. £3 13*s*. 9*d*. × 2
3. £4 16*s*. 3*d*. × 5
4. £3 9*s*. 10*d*. × 7
5. £16 8*s*. 4*d*. × 8
6. £7 0*s*. 7*d*. × 9
7. £8 14*s*. 10*d*. × 10
8. £12 11*s*. 8*d*. × 12
9. 4 ft. 8 in. × 4
10. 5 ft. 9 in. × 6
11. 1 yd. 2 ft. 7 in. × 3
12. 6 yd. 1 ft. 8 in. × 8
13. 7 yd. 2 ft. 11 in. × 9
14. 3 lb. 4 oz. × 5
15. 2 lb. 11 oz. × 11
16. 14 lb. 7 oz. × 7
17. 3 tons 4 cwt. × 6
18. 4 tons 15 cwt. 3 qtr. × 3
19. 4 tons 0 cwt. 1 qtr. × 7
20. 3 tons 19 cwt. 3 qtr. × 9

(4) DIVISION

e.g. $\dfrac{£16\ 13s.\ 1d.}{7} = £2\ 7s.\ 7d.$

NOTE

Divide into the largest units first (in this case £'s). Change the remainder into shillings and transfer it to the shillings column. Add up the shillings and divide by 7 again. Change the remainder to pence and transfer it to the pence column. Total the pence and complete.

	£	*s*.	*d*.
	2	7	7
7	16	13	1
	14	40	48
	2 ×	53	49
	20	49	49
	40	4 ×	
		12	
		48	

Such divisions do not usually work out exactly. There is a remainder which must be clearly stated with the units in the answer, as in the following example.

$\dfrac{7\text{ yd. }0\text{ ft. }11\text{ in.}}{8} = 2\text{ ft. }8\text{ in. rem. }7\text{ in.}$

	yd.	ft.	in.
		2	8 rem. 7 in.
8	7 ×	0	11
	3	21	60
	21	21	71
		16	64
		5 ×	7
		12	
		60	

Exercise 13e

1. $\frac{\text{£7 2}s.\ 3d.}{3}$

2. $\frac{\text{£14 10}s.\ 4d.}{4}$

3. $\frac{\text{£14 6}s.\ 5d.}{7}$

4. $\frac{\text{£7 2}s.\ 10d.}{8}$

5. $\frac{\text{£36 14}s.\ 3d.}{11}$

6. $\frac{\text{11 yd. 3 ft. 8 in.}}{2}$

7. $\frac{\text{37 yd. 1 ft. 1 in.}}{5}$

8. $\frac{\text{17 tons 6 cwt. 2 qtr.}}{6}$

9. $\frac{\text{18 tons 4 cwt. 2 qtr.}}{9}$

10. $\frac{\text{34 gal. 2 qt. 0 pt.}}{12}$

In some of the following examples there are remainders.

11. $\frac{\text{£16 16}s.\ 11d.}{5}$

12. $\frac{\text{£24 3}s.\ 6d.}{6}$

13. $\frac{\text{£26 8}s.\ 10d.}{9}$

14. $\frac{\text{£77 7}s.\ 1d.}{10}$

15. $\frac{\text{£51 14}s.\ 0d.}{12}$

16. $\frac{\text{11 yd. 2 ft. 7 in.}}{4}$

17. $\frac{\text{24 yd. 2 ft. 1 in.}}{7}$

18. $\frac{\text{30 stones 8 lb.}}{8}$

19. $\frac{\text{12 tons 17 cwt. 1 qtr.}}{5}$

20. $\frac{\text{14 lb. 5 oz.}}{10}$

The Rule of Twelve

This is a useful rule for rapid mental calculation.

$$1d. \times 12 = 1s.$$
$$2d. \times 12 = 2s.$$
$$3d. \times 12 = 3s. \text{ and so on.}$$

Following on: $17d. \times 12 = 17s.$

and $6\frac{1}{2}d. \times 12 = 6\frac{1}{2}s. = 6s.\ 6d.$

and $1s.\ 3\frac{1}{2}d. \times 12 = 15\frac{1}{2}d. \times 12 = 15\frac{1}{2}s. = 15s.\ 6d.$

The rule states that *to multiply pence by 12, call them shillings.*

Also $$\frac{1s.}{12} = 1d.$$
$$\frac{2s.}{12} = 2d.$$
$$\frac{3s.}{12} = 3d. \text{ and so on}$$

Hence $$\frac{14s.}{12} = 14d. = 1s.\ 2d.$$
$$\frac{17s.\ 6d.}{12} = \frac{17\frac{1}{2}s.}{12} = 17\tfrac{1}{2}d. = 1s.\ 5\tfrac{1}{2}d.$$
$$\frac{13s.\ 9d.}{12} = \frac{13\frac{3}{4}s.}{12} = 13\tfrac{3}{4}d. = 1s.\ 1\tfrac{3}{4}d.$$

To divide shillings by 12, call them pence.

Exercise 13f

1. Find the cost of 12 articles if one costs:

(a) 7*d.*	**(e)** 1*s.* 10*d.*
(b) $9\frac{1}{2}$*d.*	**(f)** 1*s.* $7\frac{1}{4}$*d.*
(c) 1*s.* 3*d.*	**(g)** 1*s.* $0\frac{1}{2}$*d.*
(d) $10\frac{3}{4}$*d.*	**(h)** 3*s.* 4*d.*

2. Find the cost of 1 article if 12 cost:

(a) 4*s.*	**(e)** 14*s.* 3*d.*
(b) 13*s.*	**(f)** 25*s.*
(c) 7*s.* 6*d.*	**(g)** 31*s.* 6*d.*
(d) 3*s.* 9*d.*	**(h)** 19*s.* 6*d.*

The Rule of Twenty

This rule is similar to the Rule of Twelve, but it is used for multiplying and dividing sums of money by twenty.

$$1s. \times 20 = £1$$
$$2s. \times 20 = £2$$
$$3s. \times 20 = £3 \text{ and so on.}$$

Following on:

$$7s. \times 20 = £7$$
$$3s.\ 6d. \times 20 = 3\tfrac{1}{2}s. \times 20 = £3\tfrac{1}{2} = £3\ 10s.\ 0d.$$

To multiply shillings by 20 call them pounds.

Also
$$\frac{£1}{20} = 1s.$$
$$\frac{£2}{20} = 2s.$$
$$\frac{£3}{20} = 3s. \text{ and so on}$$

Following on:
$$\frac{£6}{20} = 6s.$$
$$\frac{£5\ 10s.}{20} = \frac{£5\frac{1}{2}}{20} = 5\tfrac{1}{2}s. = 5s.\ 6d.$$
$$\frac{£3\ 13s.\ 4d.}{20} = \frac{£3\frac{2}{3}}{20} = 3\tfrac{2}{3}s. = 3s.\ 8d.$$

Exercise 13g

1. Find the cost of 20 articles if one costs:

(a) 12*s.*	**(e)** 12*s.* 9*d.*
(b) 17*s.*	**(f)** 3*s.* $1\frac{1}{2}$*d.*
(c) 6*s.* 6*d.*	**(g)** 6*s.* $7\frac{1}{2}$*d.*
(d) 3*s.* 4*d.*	**(h)** 2*s.* 2*d.*

2. Find the cost of one article if 20 cost:

(a) £4 10*s.*	**(e)** £3 6*s.* 8*d.*
(b) £7 5*s.*	**(f)** £4 7*s.* 6*d.*
(c) £10 15*s.*	**(g)** £6 13*s.* 4*d.*
(d) £27	**(h)** £2 17*s.* 6*d.*

Averages

The number of children present in class in a certain week were:

Monday	31
Tuesday	33
Wednesday	32
Thursday	34
Friday	33
5	163
	$32\frac{3}{5}$

Add to find the total attendances during the week. Divide this result by 5, the number of school days in the week, and obtain the result $32\frac{3}{5}$.

This is called the AVERAGE of the attendance through the week.

What does this mean? It is impossible to have $32\frac{3}{5}$ children, without serious consequences!
It means that over the week in question *about* 32 or 33 children were present each day. It is a 'middle' value for the attendance, and gives an idea of the number of children present throughout the week. If many were absent the whole week the average would be low.

Averages can be used for comparing. For example the marks of 3 children for English over 10 weeks were as follows:

John	8	7	8	4	6	7	9	10	9	8
Mary	10	8	8	7	8	10	8	7	7	8
Tom	9	7	7	6	7	6	9	8	8	8

To find the best mark in English the average mark is obtained for each pupil.

$$\text{John's average} = \frac{\text{Total marks}}{10} = \frac{76}{10} = 7{\cdot}6$$

$$\text{Mary's average} = \frac{\text{Total marks}}{10} = \frac{81}{10} = 8{\cdot}1$$

$$\text{Tom's average} = \frac{\text{Total marks}}{10} = \frac{75}{10} = 7{\cdot}5$$

Just by looking at the marks it is impossible to tell the best student, but when the averages are taken it is evident that Mary has the highest (8·1) and that there is little to choose between John (7·6) and Tom (7·5). Averages are frequently used to compare the results of football and cricket teams.

EXAMPLE

Find the average height of 4 boys whose heights are 4 ft. 3 in., 4 ft. 8 in., 3 ft. 11 in. and 4 ft. 6 in.

	ft.	in.	
	4	3	
	4	8	
	3	11	
	4	6	
4	17	4	Total
	4	4	Average

EXAMPLE

Five boys have an average age of 12 yr. 5 mo. The ages of four of them are 11 yr. 10 mo., 13 yr., 12 yr. 7 mo., 12 yr. 1 mo. Find the age of the other boy.

The total ages of the five boys = 12 yr. 5 mo. × 5
= 62 yr. 1 mo.

If total ages of the four boys = 11 yr. 10 mo. + 13 yr. + 12 yr. 7 mo. + 12 yr. 1 mo.
= 49 yr. 6 mo.

∴ The age of the other boy = 62 yr. 1 mo. − 49 yr. 6 mo.
= 12 yr. 7 mo.

Exercise 13h

1. Find the average of:

(**a**) 14, 19, 24, 18.
(**b**) 1 yd. 2 ft. 8 in., 3 yd. 1 ft. 10 in., 2 yd. 1 ft. 6 in.
(**c**) 4*s*. 6*d*., 3*s*. 2*d*., 6*s*. 2*d*., 5*s*. 3*d*., 4*s*. 3*d*.
(**d**) 1364, 2571, 8836, 1109, 4524 (answer in decimal form)
(**e**) £3 2*s*. 3*d*., £4 6*s*. 8*d*., £3 15*s*. 10*d*.
(**f**) 4 tons 3 cwt., 6 tons 13 cwt., 5 tons 10 cwt., 3 tons 13 cwt., 7 tons 1 cwt., 8 tons 14 cwt.

2. In a school there are 750 children and 32 teachers. What is the average number of children per teacher? (Give your answer as a mixed number.)

3. In a certain factory the wages bill every week is £2,156. Find the average wage if there are 176 employees.

4. A car travelled 490·5 miles and used 15 gallons of petrol. Find the average number of miles per gallon. (Give your answer in decimal form.)

5. Find the average of:

(**a**) 256, 263, 228 and 257
(**b**) 16, 28, 0, 19 and 22

6. If the average age of 30 boys is 11 years 5 months, find their total ages.

7. An author uses 123 lines to write 1,107 words. What is the average number of words per line? How many lines (to the nearest whole number) would he use in writing 1,000 words?

8. The takings in a certain shop for 6 weeks were £23 4*s*. 8*d*., £27 5*s*. 6*d*., £24 8*s*. 2*d*., £29 18*s*. 2*d*., £25 4*s*. 4*d*. and £26 6*s*. 8*d*. Calculate the average weekly takings. Use the answer to estimate (to the nearest £) the takings over a whole year.

9. In class IA 10 children scored the following marks in a test:
8, 8, 7, 5, 10, 6, 9, 8, 7, 4.
In class IB 8 children scored the following marks:
7, 7, 9, 6, 8, 7, 4, 10.
Find the average mark in each case in decimal form and say which class had the highest average.

10. A farmer sold 4 cows for £125 and on the following day sold 5 cows for £145. What was the average price per cow?

11. The average content of a box of matches is 50. A man bought 6 boxes. The contents of 5 of them were 51, 48, 49, 47, 52. How many matches would you expect the 6th box to contain?

12. A grocer mixed 7 pounds of tea at 6*s*. 2*d*. per pound with 5 pounds of tea at 6*s*. 8*d*. per pound. What was the average cost per pound of the mixture?

Exercise 13j

Miscellaneous Problems

Remember that when setting out your answers it is advisable to separate the working from the argument.

1. A householder burns 11 hundredweights of coke per month. Find the cost per month if coke costs 12*s*. 3*d*. per hundredweight. How much coke in tons and hundredweights will he burn in 6 months and how much will it cost him?

2. From a roll of carpet 30 yards long three pieces are cut. Their lengths are 31 ft. 6 in., 17 ft. 8 in. and 12 ft. 11 in. How much carpet is left? (Give your answer in yd. ft. in.)

3. A rectangular field is 37 yards wide and 102 yards long. What length of wire fencing is required to fence in all four sides? What is the cost at 6*s*. 6*d*. per yard?

4. How many quarter-pound packets of tea can be made up from one hundredweight? It is sold at 1*s*. 8*d*. per quarter pound. If one hundredweight costs the grocer £28 10*s*., what profit does he make?

5. On four days a shopkeeper takes £21 10*s*. 3d., £25 14*s*. 3*d*., £32 5*s*. 11*d*. and £19 13*s*. 8*d*. How much more must he take to make £100?

6. An electrician cuts 8 pieces of wire each 2 feet 2 inches long from a piece containing 20 feet. How much has he left over?

7. The weight of a lorry and its load is 7 tons 5 cwt. 3 qtr. If the lorry weighs 2 tons 17 cwt. 1 qtr., what is the weight of the load?

8. A sack of flour weighs 140 pounds. What is the total weight of 27 such sacks in tons, hundredweights and pounds?

9. Find the cost of 12 articles at £4 3*s*. 2*d*. each and the cost of 11 articles at £3 19*s*. 10*d*. each. Find also the total cost.

10. A father divides £4 3*s*. 6*d*. equally between his three sons. How much does each get?

11. An aircraft makes a certain flight in 5 hours 28 minutes. The return journey takes 6 hours 53 minutes. Find the total flying time and the average time for the journey.

12. A teacher took his class to the theatre. There were 30 children. The return fare was 1*s*. 9*d*. for each child and 3*s*. 6*d*. for the teacher. The theatre tickets cost 4*s*. 6*d*. each and the teacher paid the same price for both the children and himself. What was the total cost of the outing?

13. What is the total cost of 12 square yards of carpet at £3 8*s*. 6*d*. per square yard and 12 square yards of underfelt at 7*s*. 9*d*. per square yard?

14. How many pieces of string each 1 foot 7 inches long can be cut from a roll 30 yards long? How much is left over?

15. A certain type of steel girder weighs 17 pounds per foot run. Find the weight in hundredweights and pounds of a girder 23 feet long.

16. The school day begins at 9.20 a.m. and finishes at 4.30 p.m. from Monday to Friday. How many hours and minutes is it open in a week? If the school year is 39 weeks and 3 days, how many whole days and hours does a pupil spend at school?

17. A man earns £12 10*s*. for a 40-hour week. How much is he paid per hour? He is paid time and a half for each hour worked in a week more than 40 hours. If he did 53 hours in a particular week, how much wages would he get?

18. A piece of fencing has 13 posts each 6 inches wide. The distance between each post is 7 ft. 8 in. What is the total length of the fencing measured from the outside of the end posts? (Give your answer in yd. ft. in.)

19. A motor car averages 31 miles per gallon of petrol. How much petrol is required for a journey of 527 miles? What is the total cost of the petrol at 4*s.* 5*d.* per gallon?

20. Six boys have weights of 7 st. 3 lb., 8 st. 2 lb., 7 st. 11 lb., 6 st. 13 lb., 7 st. 5 lb. and 8 st. 6 lb. Find their average weight. Another boy joins the group and the new average weight is 7 st. 7 lb. What is the weight of the new boy?

14 Measuring Heights

Take the largest piece of squared paper that you can get and draw in the bottom left-hand corner a right-angled triangle ABC with base AB 6 inches and its height BC 3 inches. Now extend the base and the sloping side as far as you can across the paper as in fig. 14.1.

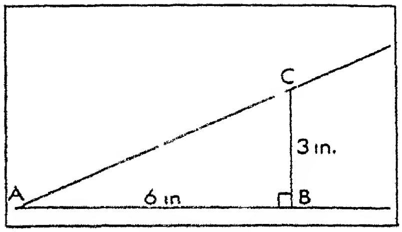


fig. 14.1

(Take a guess at the size of the angle you have drawn.)
Using the lines on your squared paper draw about ten other 'heights' B_1C_1, B_2C_2, B_3C_3, B_4C_4, etc., perpendicular to the base. We have actually constructed a number of triangles, follow them round: ABC, AB_1C_1, AB_2C_2, AB_3C_3, AB_4C_4, etc., fig. 14.2.

By counting the squares, find AB_1 and B_1C_1. Then find AB_2 and B_2C_2. Continue this for all the triangles. Make a neat table similar to that shown below in your exercise book and fill in the information required.

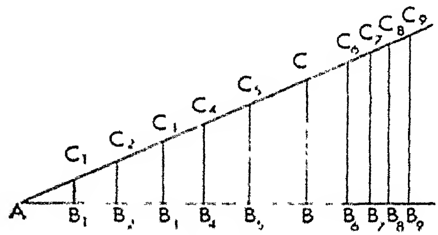


fig. 14.2

Exercise 14a

	BASE	HEIGHT	$\frac{\text{HEIGHT}}{\text{BASE}}$
ABC	6	3	$\frac{6}{3} = 2$
AB_1C_1			
AB_2C_2			
AB_3C_3			
AB_4C_4			

If you have worked accurately you will find that the answers in the last column are the same. This is rather startling since all the triangles are of different sizes. What is it then about the triangles which makes the value of $\frac{\text{height}}{\text{base}}$ always the same? The best way to see the answer to this question is to draw out several of the triangles on the back of your squared paper and look at them closely considering all the aspects of a triangle.

Have they the same area?
Have they sides of the same length?
What then have they in common?

A close look will show that the angles of all the triangles are the same. From this we can learn a most important fact: that the fraction $\frac{\text{height}}{\text{base}}$ depends NOT on the length of the sides but only upon the angle between the sloping sides.

The above fact was known several thousand years ago and was used in various ways to help men measure high objects. One method was the 'shadow' method. The height of an obelisk could be found by measuring the shadow it cast and at the same time measuring the shadow of a stick of known length. The sun's rays would make the same angle with the stick as with the obelisk and thus the height of the stick divided by the length of the stick's shadow would be the same as the height of the obelisk divided by the obelisk's shadow. Let us see how the problem would be solved. If you drew out the objects they would appear just like two of your triangles (the angles, however,

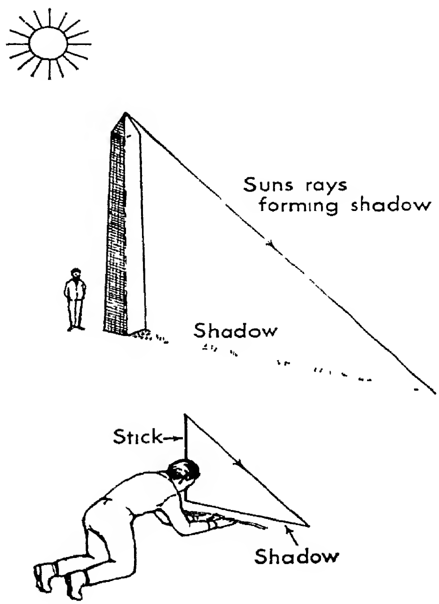


fig. 14.3

would not necessarily be the same as yours). You know that the fraction $\frac{\text{height}}{\text{base}}$ would be the same for the two triangles. This number could be obtained from the stick, for if the stick is 4 feet high and its shadow is 2 feet long the number would be $\frac{4}{2}$, i.e. 2. Then the number $\frac{\text{height}}{\text{base}}$ for the obelisk and its shadow would also be 2. Suppose the length of the shadow is 50 feet, then $\frac{\text{height}}{\text{base}} = \frac{\text{height}}{50} = 2$ and you can see that the height must be 100 feet.

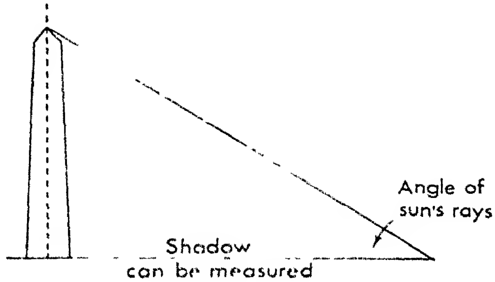


fig. 14.4

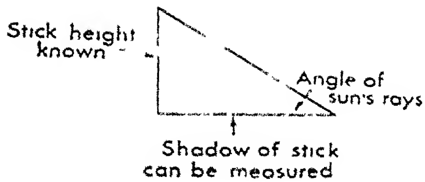


fig. 14.5

Some Work to Do

You can use the above method yourself to measure the height of a wall in the playground. Nail a stick to a small stand so that the total height is 2 feet. Wait until it is sunny and then measure the length of the shadow thrown by your stick and the length of the shadow thrown by the wall. If the length of your shadow is an odd amount like 1 foot $3\frac{1}{2}$ inches, wait a little while and the length will alter (why?). When it is an easy fraction, then measure your wall's shadow. Using the following formula work out the height of the wall:

$$\frac{\text{height of stick}}{\text{length of stick's shadow}} = \frac{\text{height of wall}}{\text{length of wall's shadow}}$$

Exercises 14b

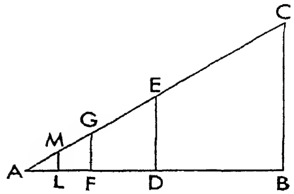


fig. 14.6

1. Find the heights in fig. 14.6 of DE, FG, and LM. When AB = 12 in., BC = 8 in., AD = 6 in., AF = 3 in., AL = $1\frac{1}{2}$ in.

2. Find the heights in fig. 17 of VW, PQ, and ST. When XY = 16 in., YZ = 16 in., XV = 9 in., XP = 6 in., XS = 3 in.

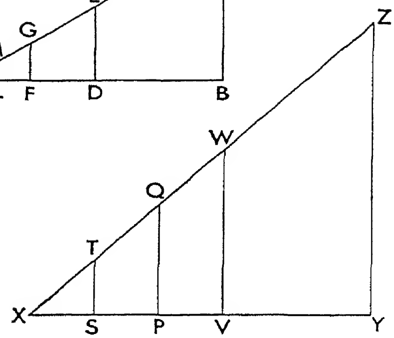


fig. 14.7

3. A boy 5 feet tall notices that the top of a wall is in line with the top of a tree when he is 10 feet from the wall. He measures the distance from the tree to the wall and finds it is 50 feet. How high is the tree if the wall is 10 feet. ? (Draw a rough diagram to help you.)

4. A builder wishes to place supports in a roof at A and B which are 10 feet from the eaves at X and Y. If the centre height is 12 feet, how long must the supports AM and BN be?

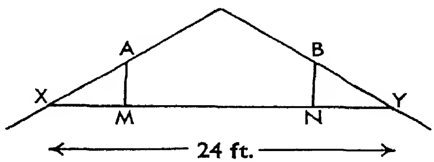


fig. 14.8

5. A ramp was needed for unloading a lorry. The floor of the lorry was 3 feet high, and the ramp base was made 9 feet long with supports placed 3 feet from each end. How high were the supports?

HOW TO MAKE A HEIGHT-FINDER

Obtain a piece of thick cardboard about a foot square. Draw very accurately on it a right-angled triangle whose height is 5 inches and whose base is 10 inches.

Continue the line of the triangle's height to the bottom of your cardboard, as in fig. 14.9. You will need two small screw-eyes which should be screwed through the cardboard and which will act as sights. They must be placed fairly accurately at the two angles of the triangle which are not 90°. A thin piece of cotton with a small weight on the end to act as a 'plumb-line' should now be tied to the top screw-eye. We are now ready to use the height-finder.

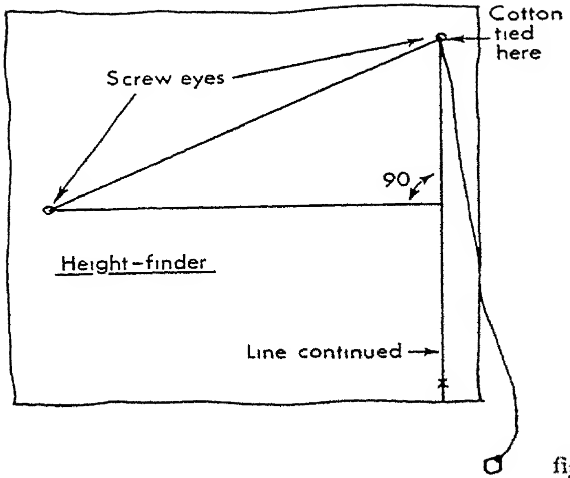


fig. 14.9

HOW TO USE THE HEIGHT-FINDER

The best method for using the height-finder is for one boy to hold the card while a second boy sights through the screw-eyes. The boy holding the card should hold it up straight so that the cotton hangs in line with the height of the triangle drawn on the card. (We extended the line to make this easier.) The second boy then sights through the screw-eyes and they move backwards or forwards until the top of the object they are measuring is seen in the sights, fig. 14.10. They then pace out or measure with a tape the distance from where they were standing to the object.

You can now find the height of the object for, as seen in fig. 14.11, the object fits into the extended sides of our triangle, thus the $\frac{\text{height}}{\text{base}}$ of the object triangle is the same as $\frac{\text{height}}{\text{base}}$ of our small triangle. This is $\frac{5}{10} = \frac{1}{2}$. Use your height-finder to find the height of the room, the height of the playground walls and buildings and of trees.

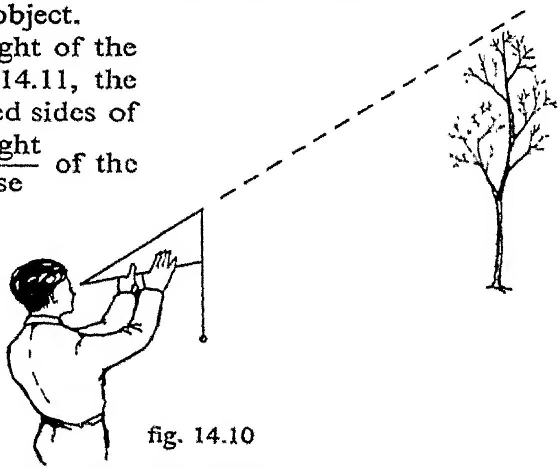


fig. 14.10

Important note: Do not forget to allow for your own height.

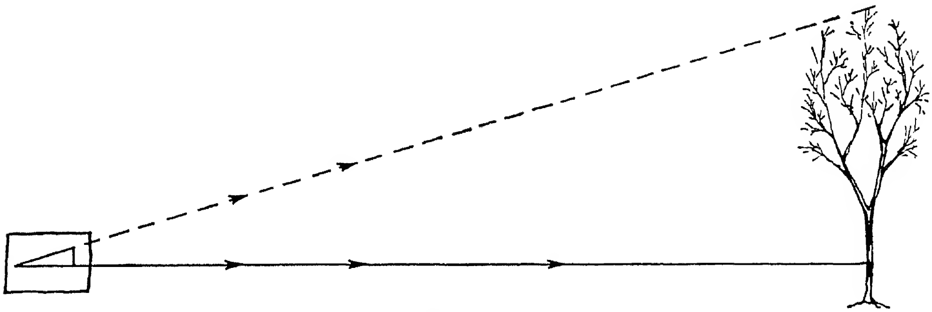


fig. 14.11

If you fit another screw-eye to your card at the spot marked X in the diagram of the height-finder and thread your cotton through it, you can if you are careful use the card without any help. You must make sure that the cotton goes through the centre of the screw-eye when you are holding it up and then you will know that your height-finder is straight and not tilted.

15 Introduction to Algebra

Problems and Missing Numbers

A fishmonger who liked to amuse his customers used to ask them this question: If a fish weighs 4 lb. and half its own weight—how heavy is it? They usually gave him the wrong answer. Have you guessed it yet? To tell the truth, he didn't know the correct answer himself until he tried it out one day on his scales.

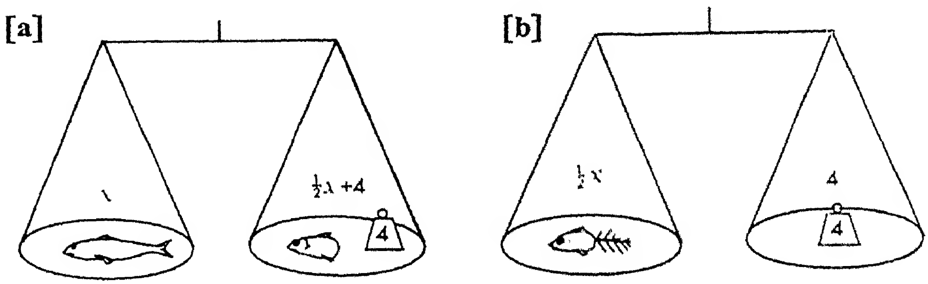


fig. 15.1

He set his scales up with the fish in one pan, and in the other pan he placed one half of another fish of exactly the same size together with one of his 4-lb. weights. Did the scales balance?
Then he took half a fish from each pan. The scales still balanced because he had taken the same weight from each pan.
He then saw the correct answer. Half the fish weighed 4 lb., and so the whole fish weighed 8 lb.
You will come across many interesting problems of this sort, and you can often find the answer by doing a practical experiment. But what could the fishmonger have done if his scales were broken?
Here is another way of doing the same problem. In detective stories the guilty person is unknown to the reader—he is often called Mr X. As the weight of the fish is unknown let us suppose it is x lb.

Then, looking at fig. [a] we could write:

$$x = \tfrac{1}{2} \text{ of } x + 4.$$

Now take $\frac{1}{2}$ of x from each side

then $\qquad \frac{1}{2}$ of $x = 4$

and hence $\qquad x = 8$

Here is another problem.

$$2 \times \bullet + 1 = 7$$

A boy made a blot in his book, and the teacher did not know at first what number was hidden by the blot. Can *you* guess?
You can use 'x' again, but it is useful to know that $2 \times x$ is written $2x$. Then

$$2x + 1 = 7$$

or $\quad x = 3$ because $2 \times 3 + 1 = 6 + 1 = 7$

You can see how this answer is obtained by looking at a pair of scales again.

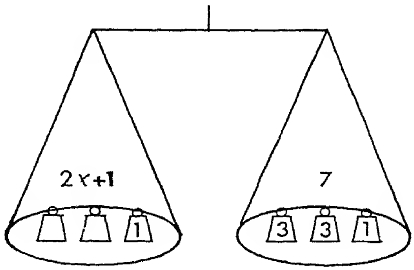


fig. 15.2

Now take a 1-lb. weight from each side

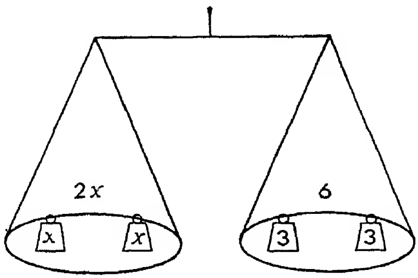


fig. 15.3

Now divide the contents of each pan by 2.

$$x = 3$$

Exercise 15a

Find the missing number.

1. $3 \times \bullet + 3 = 9$
2. $4 + 5 \times \bullet = 14$
3. $4 \times \bullet + 1 = 17$
4. $7 \times \bullet + 2 = 16$
5. $22 = 3 \times \bullet + 1$
6. $2 \times \bullet - 5 = 9$
7. $3 = 9 \times \bullet - 15$
8. $12 = 7 \times \bullet - 2$
9. $5 \times \bullet - 3 = 17$
10. $4 \times \bullet + 3 = 5$

Find the value of x in the following:

11. $4x + 1 = 9$

12. $3x + 5 = 14$

13. $4 + 2x = 18$

14. $7x - 3 = 18$

15. $12x - 1 = 23$

16. $\frac{x}{3} = 4$

17. $\frac{x}{5} = 20$

18. $16 = 3 + 13x$

19. $4 = 6x - 38$

20. $3x + 2x - 1 = 9$

21. $14 - 2x = 10$

22. $2 = 12 - 5x$

PROBLEMS

Decide carefully what 'x' is to stand for. Write down the equation and then find the value of x.

23. I think of a number, double it and add 3. The answer is 17. What was the number I first thought of? (*Hint.* Let $x =$ the number first thought of.)

24. John has x sweets. Jack has 7. If they have 12 sweets between them, how many sweets has John?

25. If I add 13 to three times x and the answer is 25, what is the value of x?

26. If twice a certain number is added to 5 the answer is 17. What is the number?

27. A piece of wood is 16 inches long. If I cut off x inches, how much is left? If the piece left is three times as long as the piece cut off, what is its length in terms of x? Write down an equation and use it to find the value of x.

28. Joan is x years old. Mary is 4 years older. What is Mary's age in terms of x? Their ages add up to 20 years. Find their ages.

29. An encyclopaedia is x inches thick. What is the total thickness of a set of 10 such books? This set is stood on a bookshelf with 3 other books, each 2 inches thick. These books take up 21 inches altogether. Find x.

30. How many pence is x shillings? Jack had x shillings, 3 sixpences, a threepenny piece and five pennies in his pocket. The total sum was 7*s*. 2*d*. Write down an equation in pence and solve it to find x.

A Geometry Problem

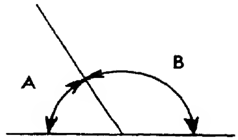


fig. 15.4

The number of degrees in angle B is twice the number in angle A. What is the size of A and B?

Let angle A be $x°$

Then angle B $= 2 \times$ angle A

$= 2x.$

But angle A + angle B $= 180°$

$\therefore x + 2x = 180°$

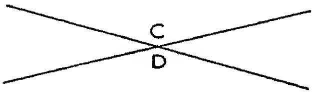
$\therefore 3x = 180°$

$\therefore x = 60°$

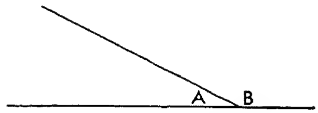
$\therefore$ A is 60° and B is 120°

Exercise 15b

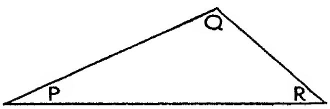
Facts to remember.



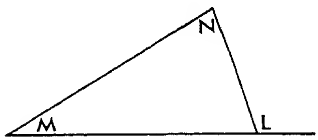
$\hat{C} = \hat{D}.$



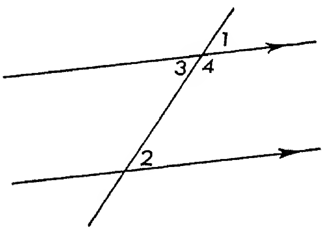
$\hat{A} + \hat{B} = 180°$



$\hat{P} + \hat{Q} + \hat{R} = 180°$



$\hat{L} = \hat{M} + \hat{N}$



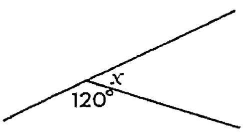
$\hat{1} = \hat{2}$

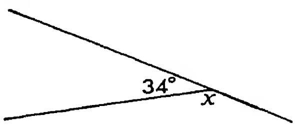
$\hat{2} = \hat{3}$

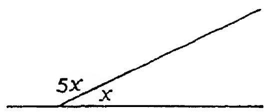
$\hat{2} + \hat{4} = 180°$

fig. 15.5

Find the angles marked x in the following figures by writing down an equation and solving it for x.







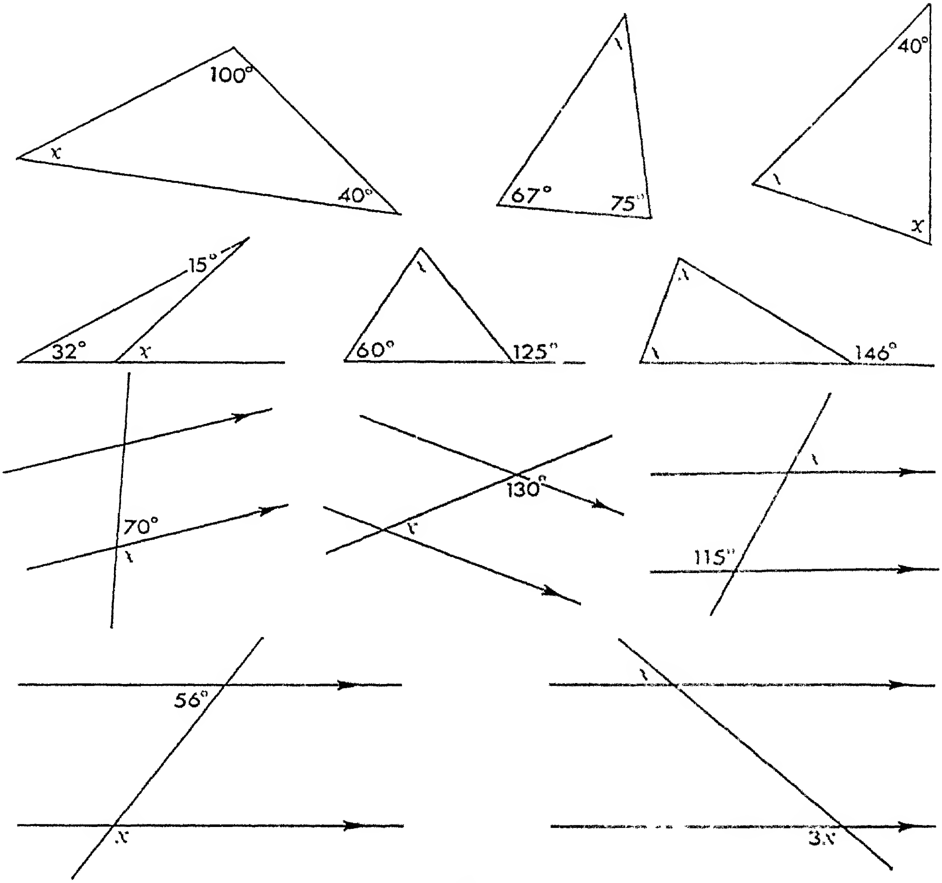


fig. 15.6

Look at this problem.

A man has two children whose names are Tom and Joan. Tom is three years older than Joan and if their ages are added together they total 25. How old are they?

The answer may be worked out by arithmetic, but it is often best to use the unknown quantity 'x'.

Let Tom be x years old.
Then Joan is $x - 3$ years old.
Their total age is 25.

$\therefore$ Tom's age $+$ Joan's age $= 25$

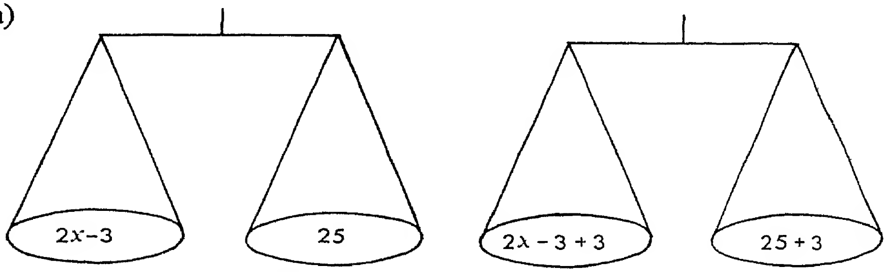
$$\therefore x + x - 3 = 25.$$

Add together the x's.

$$2x - 3 = 25$$

This expression is known as an *equation* and it is your job to find the number for x which makes $2x - 3 = 25$.
Remembering that both sides are equal you can use the idea of a pair of scales.

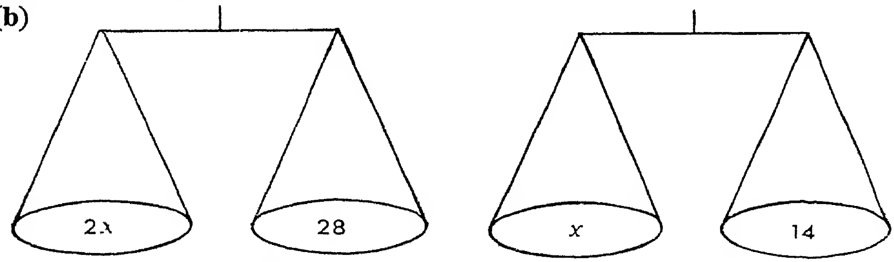
(a)



$2x - 3 = 25$ $\qquad\qquad$ $2x - 3 + 3 = 25 + 3$

Add 3 to each side

(b)



$2x = 28$ $\qquad$ fig. 15 7 $\qquad$ $x = 14$

Divide both sides by 2

It would be a long job, if every time you had an equation you had to imagine a pair of scales. You can do without them if you remember that with an equation, what you do to one side you must do to the other.

e.g. Find the value of x if

$$3x + 4 = 7$$

Take 4 from each side

$$3x + 4 - 4 = 7 - 4$$
$$\therefore 3x = 3$$
$$\therefore x = 1$$

It is useful to check your answers. 'LHS' is short for left-hand side. You write this down again replacing x by 1.

Check If

$$x = 1$$
$$\text{LHS} = 3 \times 1 + 4$$
$$= 3 + 4$$
$$= 7$$
$$\text{RHS} = 7$$
$$\therefore \text{LHS} = \text{RHS}$$
$\therefore x = 1$ is the correct solution.

Here is a more difficult example.

Solve and check: $5x - 2 = 3x + 4$

You want to finish up with a value for x and so you must get all the x's on one side and all the numbers on the other. Get the numbers over on the RHS first.

$$5x - 2 = 3x + 4 \qquad \text{(A)}$$

Add 2 to each side

$$5x - 2 + 2 = 3x + 4 + 2$$

or

$$5x = 3x + 6$$

Take $3x$ from each side

$$5x - 3x = 3x - 3x + 6$$
$$\therefore 2x = 6$$
$$\therefore x = 3$$

Check If

$$x = 3$$
$\text{LHS} = 5 \times 3 - 2$, from line (A)
$$= 15 - 2$$
$$= 13$$
$$\text{RHS} = 3 \times 3 + 4$$
$$= 9 + 4$$
$$= 13$$
$$\therefore \text{LHS} = \text{RHS}$$
$\therefore x = 3$ is the correct solution

Exercise 15c

Solve and check the following equations, remembering that other letters than 'x' may be used for the unknown quantity.

1. $3x + 1 = 13$
2. $4a = 15 - a$
3. $4y - 2y = 3 - y$
4. $3x + 5 = x + 15$
5. $12z - 30 = 2z$
6. $7 + c = 4 + 2c$
7. $16 - w = 3w + 4$
8. $a + 2a + 3a + 4a = 60$
9. $3d - 6 = d + 8$
10. $6v + 4 = 36 - 2v$

More Problems

(1) On his way to work a man travels 20 miles by bus and train. He travels three times as far on the train as he does on the bus. How far does he travel by each vehicle separately?

Read this question carefully. It contains two parts. One part contains information and the other part is the question itself. Pick out the question part and decide what your unknown x is going to stand for. In this case the distance travelled by bus and train are both required.

Let x miles = distance travelled by bus

Having decided this, the information part of the question must be used. The total distance cannot be brought in until the distance travelled by train is decided. This is three times the distance travelled by bus.

$\therefore$ $3x$ miles = distance travelled by train

But distance travelled by bus + distance travelled by train = 20 miles.

$$\therefore x + 3x = 20$$

Add the x's

$$4x = 20$$

$$x = 5$$

Now write the answer down in words.

$\therefore$ Distance travelled by bus = 5 miles
and distance travelled by train = $5 \times 3 = 15$ miles

It is always well to check answers wherever possible. It is no use in problem work just to check that $x = 5$ is the correct solution to the equation because the equation may be wrong. It is necessary to go back to the problem itself.

Check Total distance gone = $5 + 15 = 20$ miles.

This agrees with the problem and so the answers are correct.

(2) The following problem is done without any explanation.

The perimeter of a rectangle (the distance round) is 48 inches. If the bigger side is 4 inches longer than the shorter side, find the lengths of each.

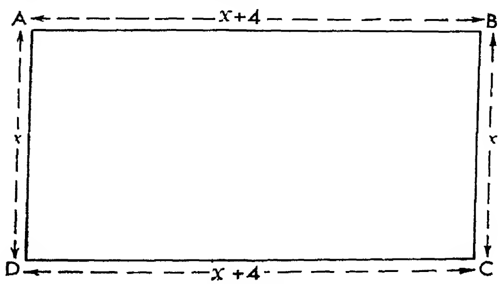


fig. 15.8

Let AD $= x$ in.
Then AB $= x + 4$ in.
$\therefore$ BC $= x$ and DC $= x + 4$ in.
Now AD + AB + BC + DC = 48 in.
$\therefore x + x + 4 + x + x + 4 = 48$
Add the x's
$$4x + 4 + 4 = 48$$
Add the numbers
$$4x + 8 = 48$$
Take 8 from each side
$$4x = 40 \quad \therefore x = 10$$
The lengths of the sides are 10 in. and 14 in.

Check Perimeter $= 10 + 14 + 10 + 14$
$= 48$ inches

Exercise 15d

Solve the following problems and check your answers.

1. Jack and Mary have £1 between them. Jack has three times as much as Mary. How many shillings have they each? (*Hint.* Let Mary have x *s*. How many shillings has Jack?)

2. The perimeter of a triangle is 24 inches. One side is 10 inches long. The second side is 4 inches shorter than the third side. Find the length of the third side. (*Hint.* Let the third side be x in. long.)

3. A man is four times as old as his son. If their combined ages are 45, what are their ages?

4. In a triangle ABC, angle A is 20 degrees more than angle C and angle B is twice angle C. Find angle C and the other two angles. (Check that your three answers add up to 180°.)

5. Share 2 shillings between two boys so that one has five times as much as the other.

6. I think of a number. When I subtract twice this number from 49 I obtain the same answer as when I multiply the number by 5. What is the number?

7. Three consecutive numbers add up to 36. Find them. (Consecutive numbers are numbers written in order, e.g. 3, 4, 5.)

8. Three-quarters of a number is 27. Find the number by means of an equation.

9. A boy weighs 4 stones and one-third of his own weight. How heavy is he? (*Hint.* Look at the beginning of the chapter.)

10. A school has 450 pupils. There are 3 boys to every 2 girls. How many boys and girls are there? (*Hint.* Let there be $2x$ girls.)

Formulae

A formula (formulae means more than one formula) is a quick way to write down a result.
For example, there are two formulae in connection with the rectangle.

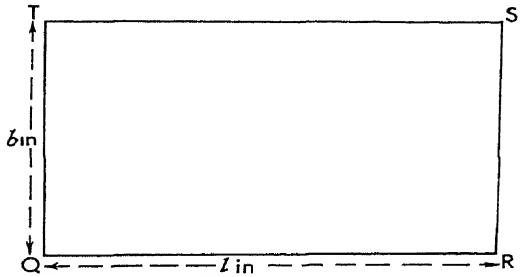


fig. 15.9

Here is a rectangle whose length is l in. and whose breadth is b in. The distance all the way round a rectangle is called its PERIMETER. Let the perimeter of the rectangle be P in.

$$\begin{aligned} \text{Then the perimeter } P &= \text{TQ} + \text{QR} + \text{RS} + \text{ST} \\ &= B + L + B + L \\ P &= 2B + 2L \end{aligned}$$

This can be written $P = 2(B + L)$
i.e. The perimeter is twice $(B + L)$.

Here are two examples to show how this formula can be used:

(1) Find the perimeter of a rectangle whose length is 4 in. and whose breadth is 3 in.

Here $\quad L = 4 \quad \text{and} \quad B = 3$

Now
$$\begin{aligned} P &= 2(B + L) \\ &= 2(3 + 4) \\ &= 2 \times 7 \\ &= 14 \end{aligned}$$

$\therefore$ Perimeter of rectangle is 14 in.

(2) The perimeter of a rectangle is 20 in., its length being 6 in. Find its breadth.

Here $\quad L = 6;\ B = \ ?;\ P = 20$

Now
$$\begin{aligned} P &= 2(B + L) \\ \therefore 20 &= 2(B + 6) \end{aligned}$$

This is an equation for B which is solved in the usual way.

Divide both sides by 2
$$10 = B + 6$$
Take 6 from both sides
$$\therefore B = 4$$
Breadth of rectangle is 4 in.

In Chapter 11 it was shown that the area of this rectangle was length $\times$ breadth, giving an answer in square inches.
If the area is now called A sq. in. the calculation of the area can be written:
$$A = L \times B$$
or, in short, $$A = LB.$$
Find the area of a rectangle 5 in. in length and 3 in. in breadth.
$$L = 5 \text{ and } B = 3$$
$$\begin{aligned} A &= LB \\ &= 5 \times 3 \\ &= 15 \text{ sq. in.} \end{aligned}$$
The area of a rectangle is 28 sq. in. It is 4 in. wide. How long is it?
$$A = 28; \quad B = 4; \quad L = ?$$
$$A = LB$$
$$\therefore 28 = L \times 4$$
i.e. $$4L = 28, \text{ another equation,}$$
or $$L = 7$$
The length of the rectangle is 7 in.

NOTE
When a formula is used directly it gives the required answer straight away.
When it is used indirectly it gives an equation which has to be solved to give the required answer.

Exercise 15e

The following examples use the formula for the perimeter of a rectangle obtained above: $P = 2(B + L)$.

1. Find P when $B = 3$ in., $L = 4$ in.
2. Find P when $B = 2\frac{1}{2}$ ft., $L = 5$ ft.
3. Find P when $L = 4\frac{3}{4}$ yd., $B = 1\frac{1}{4}$ yd.
4. Find P when $B = 3{\cdot}5$ in., $L = 4{\cdot}75$ in.
5. Find L if $P = 8$ in., $B = 1$ in.

6. Find B if $P = 16$ in., $L = 5$ in.
7. Find B if $P = 60$ ft., $L = 25$ ft.
8. Find L if $P = 27$ in., $B = 3\frac{1}{2}$ in.
9. Find L if $P = 13$ in., $B = 2{\cdot}7$ in.
10. Find B if $P = 15$ in., $L = 4\frac{3}{4}$ in.

The following examples use the formula for the area of a rectangle: $A = LB$.

11. Find A if $L = 10$ in., $B = 3$ in.
12. Find A if $L = 3\frac{1}{2}$ in., $B = 2\frac{1}{2}$ in.
13. Find A if $B = 3{\cdot}7$ in., $L = 4{\cdot}8$ in.
14. Find B if $A = 27$ sq. yd., $L = 9$ yd.
15. Find B if $A = 16$ sq. ft., $L = 12$ ft.
16. Find L if $A = 22{\cdot}5$ in., $L = 15$ in.
17. Find L if $A = 3$ sq. in., $B = \frac{4}{5}$ in.
18. Find B if $A = 18$ sq. ft., $L = 13\frac{1}{2}$ ft.

The following examples use the formula $v = u + ft$.

19. Find v if $u = 3, f = 6$ and $t = 2$.
20. Find v if $u = 3{\cdot}6, f = 0{\cdot}5$ and $t = 2{\cdot}6$.
21. Find u if $v = 24, f = 5$ and $t = 3$.
22. Find u if $v = 9, f = \frac{1}{2}$ and $t = 6$.
23. Find f if $v = 14$, $u = 5$ and $t = 3$.
24. Find f if $v = 46$, $u = 20$ and $t = 13$.
25. Find t if $v = 19$, $u = 4$ and $f = 3$.
26. Find t if $v = 21{\cdot}3$, $u = 9{\cdot}6$ and $f = 5$.

27. If $l = \dfrac{L}{1 + Tx}$

(a) Find l if $T = 8$, $x = 3$ and $L = 200$.
(b) Find x if $L = 40$, $l = 5$ and $T = 14$.
(c) Find L if $l = 100$, $T = \dfrac{1}{4}$ and $x = 16$.
(d) Find T if $L = 36$, $l = 4$ and $x = \dfrac{1}{2}$.

28. If $x = \dfrac{W}{W + w} \times d$

(a) Find x if $W = 14$, $w = 4$ and $d = 9$.
(b) Find d if $x = 25$, $W = 5$ and $w = 2$.
(c) Find w if $x = 9$, $d = 36$ and $W = 3$.
(d) Find W if $w = 6$, $x = 4$ and $d = 8$.

16 Revision Exercises

These exercises may be used for revision purposes or as test papers. The first six exercises are based on chapters 1–6.

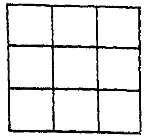


fig. 16.1

Exercise 1

1. How many squares are there in the diagram?

2. What fraction of a revolution are the following angles? **(a)** 90° **(b)** 180° **(c)** 60° **(d)** 120° **(e)** 45°.

3. A football pitch measures 100 yards by 75 yards. Using a scale of 1 in. = 25 yd. draw a diagram of the pitch. How much shorter is it to walk diagonally across the pitch than to walk round two sides?

4. 87 ÷ 3; 216 ÷ 4; 440 ÷ 8; 198 ÷ 9; 168 ÷ 7; 258 ÷ 6; 3396 ÷ 12; 1012 ÷ 11; 215 ÷ 5; 294 ÷ 7.

5. 24 × 16; 49 × 24; 117 × 42; 23 × 115; 35 × 118; 69 × 22; 98 × 41; 145 × 17; 243 × 25; 410 × 36.

6. If one box contains 42 matches, how many matches are there in a gross of boxes? If the manufacturer puts two fewer in each box how many does he save in a gross of boxes? How many boxes would these matches fill?

	13	
7	17	

fig. 16.2

Exercise 2

1. Copy out and complete the magic square. Each line should add up to 39.

2. On a scale of 1 in. = 1 ft., what would the following lengths represent? **(a)** 3 in. **(b)** $\frac{1}{2}$ in. **(c)** $2\frac{1}{4}$ in. **(d)** $5\frac{3}{4}$ in. **(e)** $2\frac{1}{8}$ in.

3. How many degrees does the hour hand of a clock move through in $\frac{1}{2}$ hour? How many degrees does the minute hand of a clock move through in $\frac{1}{2}$ hour? At 12 o'clock the hands of a clock are together, what is the angle between them at 12.30?

4. What is the difference between 23 and 48? 43 and 37? 17 and 135? 286 and 77? 589 and 978?

5. Add across:
   (a) $14 + 38 + 24 + 16 + 27 + 89$
   (b) $99 + 71 + 83 + 94 + 31 + 2 + 16$
   (c) $54 + 45 + 36 + 63 + 85 + 58$

6. A country postman cycles 96 miles each week. Allowing for two weeks' holiday per year, how many miles does he cover if he does the same round for 6 years?

Exercise 3

1. What is the angle between the hands of a clock at (a) 4 a.m. (b) 4.30 a.m. (c) 2.00 a.m. (d) 2.15 a.m. NOTE: look at number 3 problem in exercise 2 first.

2. The foot of a ladder is 9 feet from a wall, and the top just reaches a window 12 feet high. By drawing to a scale of $\frac{1}{2}$ inch equal to 1 foot, find the length of the ladder.

3. Three successive numbers add up to 36. What are they?

4. Outside the observatory at Greenwich there is a 24-hour clock. Draw accurate diagrams to show the positions of the hands at (a) 9.40 a.m. (b) 5.20 p.m.

5. At an election one candidate received 1,712 votes more than his opponent. If the total votes cast was 23,482, how many votes did each candidate receive?

6. Tom has 16 marbles, his friend John has 23 and his brother Mark has 36. How many marbles must Mark give to Tom and John if they want to start a game with the same number of marbles each?

Exercise 4

1. A gate is $7\frac{1}{2}$ feet long and 4 feet high. Find, by drawing, the length of a strut to go from one corner to the opposite corner.

2. Clues for the number square:
   1 across: 17×17
   2 across: $47 + 89 + 33 + 65 + 23 - 68$
   3 across: $1911 \div 13$

By adding up the numbers in the squares you can check your answers.
It should come to 49.

1		
2		
3		

fig. 16.3

3. A boy had a 4-inch cube painted red. He decided to cut it into

1-inch cubes as in the diagram. How many did he get? How many of the cubes were painted on three sides, how many painted on two sides, how many on one side and how many were not painted at all?

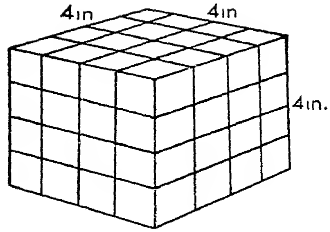


fig. 16.4

4. In the addition sum shown here the ink was smudged where there is an asterisk. If the sum was correct, write it out fully putting in the missing digits.

$$\begin{array}{r} *\,1\,3\,9 \\ 7\,* \\ 9\,7\,7 \\ 4\,*\,0\,5 \\ \hline 7\,4\,*\,5 \\ \hline \end{array}$$

5. One of the angles of a triangle is given, and the other two angles in the triangle are equal. What would the equal angles be if the given angle is (a) 40° (b) 50° (c) 64° (d) 90° (e) 156°.

6. What is the least number of coaches to take 435 boys on an outing if each coach takes only 34?

Exercise 5

1. Draw six squares, each of side $2\frac{1}{2}$ in. In the first three squares copy diagrams a, b and c. In the 4th square draw BOTH a and b. In the 5th square draw BOTH a and c. In the last square draw both b and c.

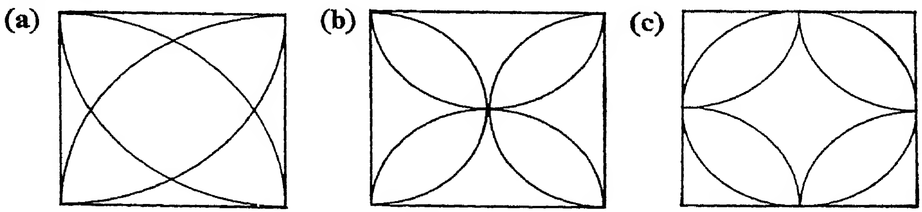


fig. 16.5

2. How many triangles are there in fig. 16.6?

3. If 25,805 bushes are planted on 65 acres, how many is this to the acre?

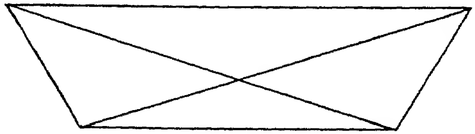


fig. 16.6

4. A man starts from point A and walks 5 miles in a direction N20°E to a point B; he then walks N80°E to C. Find from a scale drawing the distance and bearing of C from A, if BC is 4 miles.

5. A man writes on an average 13 words to a line. If there are 23 lines per page how many words will he have written if his story has filled 254 pages?

6. What is the difference between the greatest number and the least number which can be made from the digits 5, 3, 9, 2, 7?

Exercise 6

1. A man cuts 4 pieces as in fig. 16.7 from a plank. If there was no wasted material at all, how long was the plank?

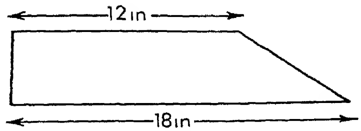


fig. 16.7

2. A man drove 12,500 miles in a year and used 446 gallons of petrol. How many miles does the car do to a gallon?

3. A, B, C are three church towers; B is 10 miles north of A, while C is 15 miles from A in a direction N35°W. Find by a scale drawing (i) the distance from B to C (ii) the bearing of C from B.

4. In fig. 16.8 find angles a, c, d, e, f, when b = 72°, h = 105°, and g = 128°.

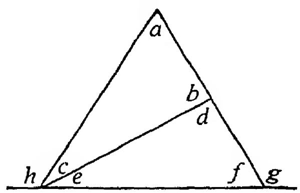


fig. 16.8

5. In 22 boxes each containing 36 dozen eggs only 39 were broken. How many were unbroken?

6. Find three consecutive numbers which when multiplied together give 39,270. Use trial and error.

Exercise 7

An exercise on series.
Look at the numbers in each line. Find how they change then write down the two numbers that should come next.

1. 1, 3, 5, 7, 9, 11, . . .
2. 2, 5, 8, 11, 14, 17, . . .
3. 1, 2, 4, 7, 11, 16, . . .
4. 1, 3, 7, 13, 21, 31, . . .
5. 55, 46, 37, 28, 19, . . .
6. 55, 46, 38, 31, 25, 20, . . .
7. 7, 12, 17, 22, 27, 32, . . .
8. 128, 64, 32, 16, 8, 4, . . .
9. 12, 1, 11, 2, 10, 3, 9, 4, 8, 5, . . .
10. 2, 3, 4, 3, 4, 5, 4, 5, 6, 5, 6, . . .

Exercise 8

Find the wrong numbers in the following series.

1. 1, 3, 5, 7, 10, 11, 13, 15.
2. 1, 4, 9, 16, 20, 25, 36.
3. 91, 84, 77, 71, 63, 56, 49.
4. 8, 4, 2, 1, 0, $\frac{1}{2}$, $\frac{1}{4}$.
5. 1, 3, 6, 10, 14, 21, 28.
6. 2, 4, 8, 16, 30, 64, 128.
7. 12, 1, 11, 2, 10, 3, 8, 4.
8. 729, 243, 81, 26, 9, 3.
9. 100, 50, 10, 90, 50, 20, 80, 60, 30, 70.
10. 240, 176, 144, 126, 120.

Exercise 9

1. Find the weight of a wooden beam which is 9 in. square and 12 ft. long if 1 cu. ft. weighs $42\frac{1}{2}$ lb.

2. How many triangles can you find in the diagram? Work through systematically by size.

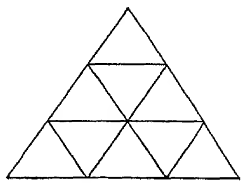


fig. 16.9

3. What length of paper 1 in. wide can be cut from 1 sq. ft. of paper? What length of paper 1 in. wide can be cut from 2 sq. ft. of paper? What length of paper 1 in. wide can be cut from a piece of paper which is 2 foot square? (Draw diagrams to help.)

4. The volume of a pyramid is given by the formula $\dfrac{L \times B \times H}{3}$.

Find the volumes of the following:
(a) L = 5 in., B = 6 in., H = 8 in.
(b) L = 7 in., B = 9 in., H = 2 in.
(c) L = 8 in., B = 2 in., H = 1 ft.
(d) L = 3 ft., B = 4 ft., H = 6 in.
(e) L = 6 in., B = 9 in., H = 2 ft.

5. (a) $\frac{1}{4} - \frac{1}{8}$ (b) $\frac{1}{2} - \frac{3}{8}$ (c) $\frac{3}{4} - \frac{1}{8}$ (d) $\frac{3}{4} - \frac{3}{8}$ (e) $\frac{3}{4} - \frac{5}{8}$ (f) $\frac{7}{8} - \frac{3}{4}$ (g) $\frac{5}{8} - \frac{1}{2}$ (h) $\frac{3}{8} - \frac{1}{4}$ (i) $\frac{7}{8} - \frac{1}{2}$ (j) $\frac{5}{8} - \frac{1}{4}$.

6. A man is paid 6*s.* 9*d.* per hour, and time and one-third for overtime. Find his earnings in a week of 42 hours plus 6 hours overtime.

Exercise 10

1. An acre plot is 121 yd. wide. How long is the plot if it is rectangular in shape?

2. $\frac{3}{8} \times \frac{5}{12}$, $\frac{11}{12} \times \frac{3}{7}$, $\frac{7}{8} \times \frac{16}{19}$, $\frac{4}{5} \times \frac{9}{16}$, $\frac{3}{4} \times \frac{4}{9}$, $\frac{4}{15} \times \frac{5}{16}$, $\frac{5}{8} \times \frac{12}{25}$, $\frac{8}{9} \times \frac{15}{48}$, $\frac{9}{20} \times \frac{16}{21}$, $\frac{12}{35} \times \frac{15}{16}$.

3. Solve these equations: $x + 3 = 6$, $x + 5 = 9$, $x + 8 = 19$, $x + 23 = 31$, $x - 2 = 5$, $x - 4 = 6$, $x - 7 = 16$, $x - 11 = 5$, $x + 3x = 12$, $7x + 11x = 90$.

4. A man had 120 oranges which he was to sell at 5 for 2*d.* As there were large and small he decided to divide them into two equal lots. One lot he sold at 3 for 1*d.*, and the others he sold at 2 for 1*d.* How much would he have got? How much did he get? What is the better way to sell them? What was the difference?

5. Write down the vulgar fractions corresponding to the following percentages: 20%, $12\frac{1}{2}$%, 5%, $2\frac{1}{2}$%, $37\frac{1}{2}$%, $33\frac{1}{3}$%, $66\frac{2}{3}$%, 15%, 35%, 160%.

6. A 4-foot stick throws a shadow 5 feet long; what length shadows are thrown by a wall 12 feet high and a pole 10 feet high?

Exercise 11

1. Boulogne is 26 miles S24°E of Dover, while Calais is 21 miles S67°E of Dover. Find the distance and bearing of Boulogne from Calais.

2. Solve these equations: $3x - 5 = 7$, $4x - 9 = 27$, $2x - 13 = 29$, $7x - 11 = 31$, $4x = 14 - 3x$, $6x = 40 - 4x$, $8x = 32 + 4x$, $17x = 40 + 9x$, $3x - 5 = 2x + 3$, $9x - 9 = 26 + 4x$.

3. One cubic foot of copper weighs 540 lb. Find the weight of a $\frac{1}{4}$-in. thick sheet measuring 4 ft. 6 in. by 1 ft. 6 in.

4. A workman's wages for the four weeks during February were £17 17*s.* 0*d.*; £18 19*s.* 8*d.*; £18 14*s.* 0*d.*; and £20 2*s.* 4*d.*

- **(a)** What was his average weekly wage?
- **(b)** If his 'basic' time was 42 hours at 8*s.* 6*d.*, how much would he earn if he did not work overtime?
- **(c)** If his overtime rate was time and a third, how much would he earn for each hour of overtime?
- **(d)** How much did he earn on overtime during February?
- **(e)** How many hours did he work overtime during this month?

5. Draw a rectangle $4\frac{1}{2}$ in. $\times$ 3 in. Inside the rectangle draw carefully six circles, as in the diagram, opening your compasses to $\frac{3}{4}$ in. The circles should not cut each other, but should just touch.

6. A cake weighed 6 lb. 4 oz. At a guessing competition a woman guessed 7 lb. 2 oz., and her daughter guessed 5 lb. 7 oz. Which was the better guess?

Exercise 12

1. A factory floor is 75 ft. $\times$ 140 ft. In fitting various machines it was found that including alleyways each machine needed an average of 35 sq. ft. How many machines were installed?

2. Solve the following equations:

(a) $3 + 6x = 4x + 5$
(b) $10x - 17 = 7 - 2x$
(c) $4y - 18 = 8y + 2$
(d) $5n - 25 = 2n - 1$
(e) $4 - 17s = s - 5$
(f) $8 - 6t = 5t$
(g) $19p - 23 = p - 14$
(h) $3k - 5 = 2k + 11 - k$
(i) $4k - 11 - 3k - 7 + k = 0$
(j) $5h - 8 + 3h = 6h + 2$.

3. Find all the numbers that will divide into 829 and give a remainder of 24. (There are 6 numbers.)

4. Supply the missing digits in the following subtraction sums:

(a)
```
 8*74* −
  6*23
 ─────
 *11*6
 ─────
```

(b)
```
 5483* −
  1*2**
 ─────
 *9639
 ─────
```

5. (a) $1\frac{1}{4} \div \frac{2}{3}$ (b) $\frac{7}{8} \div \frac{2}{3}$ (c) $\frac{3}{4} - \frac{2}{3}$ (d) $1\frac{1}{2} \div \frac{3}{4}$ (e) $2\frac{1}{2} \div \frac{3}{5}$ (f) $1\frac{1}{3} \div \frac{2}{5}$ (g) $2\frac{1}{2} \div \frac{5}{16}$ (h) $1\frac{1}{2} \div \frac{3}{8}$ (i) $8\frac{1}{2} \div 3\frac{3}{16}$ (j) $2\frac{1}{4} \div 1\frac{3}{4}$

6. A carpet is laid in a room 15 ft. $\times$ 11 ft. 6 in., leaving a border 2 ft. 6 in. wide all round. What is the area of the carpet?

Exercise 13

1. Draw a rectangle ABCD where AB 2 in. and AD 1·5 in. Join AC. Using a set square, draw a line from B to AC perpendicular to AC. If this line meets AC at N, measure AC and BN.

2. A boy notices that when the top of a 10 ft. wall is in line with the top of a tower he is 9 ft. from the wall. He finds out that the tower is 80 ft. high; allowing for his own height of 5 ft., how far was the wall from the tower?

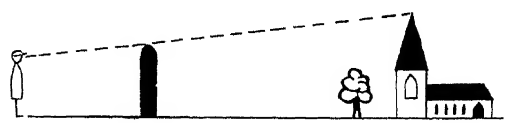


fig. 16.10

3. The sum of three numbers is 40. The second number is twice the first, and the third number is five larger than the second. Find the numbers. (Let the first number be x.)

4. Find angle a in the diagram.

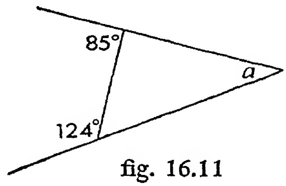


fig. 16.11

5. (a) $2{\cdot}3 \times 1{\cdot}5$ (b) $2{\cdot}6 \times 1{\cdot}8$ (c) $3{\cdot}9 \times 1{\cdot}2$ (d) $2{\cdot}2 \times 2{\cdot}7$ (e) $4{\cdot}3 \times 2{\cdot}7$ (f) $3{\cdot}9 \times 2{\cdot}5$ (g) $5{\cdot}7 \times 3{\cdot}1$ (h) $6{\cdot}5 \times 4{\cdot}3$ (i) $8{\cdot}7 \times 4{\cdot}6$ (j) $7{\cdot}4 \times 3{\cdot}9$.

6. The gas man took 297 coins from the gas meter. If they were only sixpences and shillings, and there were twice as many sixpences as shillings, how much was there in the meter?

Exercise 14

Across 1. A square number.
4. Find x, if $2x - 5 = 53$.
5. Find the average of $45 - x$, 40, and $x - 1$.
6. The largest 3-figured number, when all the digits are different.
8. 41×17.
10. Find angle x.

Down 1. Five 2's multiplied together.
2. $(7 \times 25 \times 4 \times 5 \times 2) - 1$.
3. Find y.
5. $4x + 5y$, when $x = 2$ and $y = 4$.
7. $(a - b) \times (a + b)$, when $a = 40$, $b = 27$.
8. How many threepenny pieces in 15*s*. 6*d*.?
9. A number in a team.

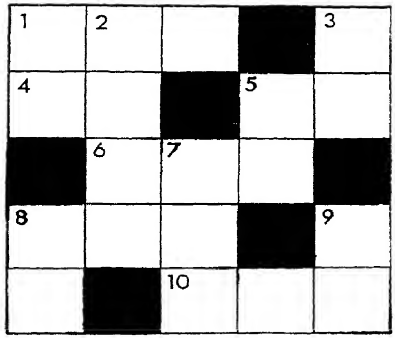


fig. 16.12

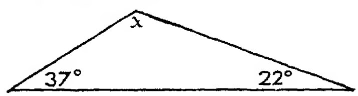


fig. 16.13

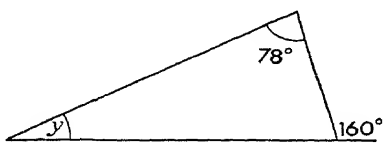


fig. 16.14

Exercise 15

1. Find the missing numbers in this subtraction.

```
*271 —
5*9*
37*6
```

2. A brick weighs one pound and half a brick. What does the brick weigh? (Let the weight be x lb.)

3. The volume of a sphere is given by vol $\frac{88}{21} \times \frac{R}{1} \times \frac{R}{1} \times \frac{R}{1}$, where R is the distance from the centre to the outside.
Find the volumes of:

(a) a tennis ball, where $R = 2\frac{1}{2}$ in.
(b) a football, where $R = 4\frac{1}{4}$ in.
(c) a strato-balloon, where $R = 5\frac{1}{4}$ ft.

4. A trench 90 yd. long, $2\frac{1}{2}$ ft. wide, 5 ft. deep is to be dug. How many lorry loads will this be if a lorry can take 12 cubic yards of dug earth?

5. In the diagram find angle e. Draw out the figure on squared paper, with AP 3 in. and PQ $3\frac{1}{2}$ in. If AP points due north–south, what is the bearing of Q from R?

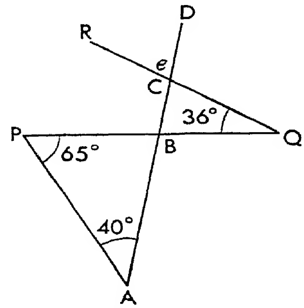


fig. 16.15

6. Find the cost of:
$3\frac{3}{4}$ lb. of tea at 1*s*. 8*d*. per qtr.
$4\frac{1}{4}$ lb. of coffee at 1*s*. 10*d*. per qtr.
$1\frac{1}{2}$ lb. of butter at 2*s*. 10*d*. per lb.
$2\frac{1}{4}$ lb. of cheese at 3*s*. 9*d*. per lb.

Exercise 16

1. A cycle costs £28 cash or 12 monthly payments of £2 14*s*. How much extra is paid if purchase is made by instalments? What per cent. has been added to the cost price?

2. Two boats leave port together, one proceeds N15°E at 18 m.p.h., the other N80°W at 12 m.p.h. Find by drawing how far apart they are after one hour.

3. A rectangle has one side 4 feet shorter than the other. If the perimeter is 12 yards, find the length of the sides.

4. A cube has edges of length $\frac{a}{3}$ in.

(a) What is the area of one face?
(b) What is the total surface area?
(c) What is the total length of all the edges?
(d) What is the volume of the cube?

5. A car travelling at an average speed of 36 m.p.h. completes a journey in $2\frac{1}{2}$ hours. If the average speed had been only 30 m.p.h., how long would the journey have taken?

6. An important fact to learn is that 1 cu. ft. of water = $6\frac{1}{4}$ gallons. How many gallons will a cistern contain that measures 6 ft. × 4 ft. × $2\frac{1}{2}$ ft.?
How many gallons in 100 cu. ft.?
How many cu. ft. in 100 gallons?

Exercise 17

1. (a) 36 + 174 + 3598 + 275 (b) 3·6 + 1·74 + 35·98 + 0·275

2. Find the difference between
(a) 988 and 106
(b) 0·988 and 1·06
(c) £3 12*s.* 8*d.* and £4 6*s.* 1*d.*

3. Find the area of the room in sq. ft. How many square yards of lino will be needed to cover the room? (Give your answer to the nearest sq. yd.) Find the cost of the lino at 18*s.* 6*d.* per sq. yd.

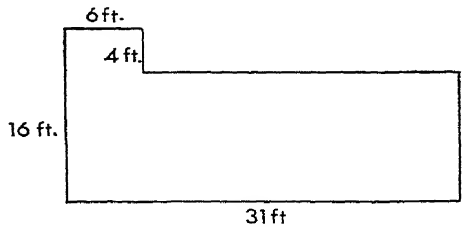


fig. 16 16

4. Calculate angles ACD and ACB. Find also angle ACX.

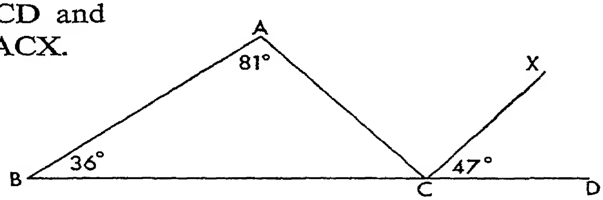


fig. 16.17

5. Find the average weight of a group of six boys whose weights are, 8 stone 7 pounds, 7 st. 13 lb., 7 st. 4 lb., 9 st. 1 lb., 8 st. 10 lb., 8 st. 3 lb.
Another boy of weight 6 st. 11 lb. joins the group. What is the new average weight?

6. In a box of oranges 37% were bad. If there were 441 *good* oranges, find how many the box contained originally. If the good ones were sold at 9 for 2*s.* 6*d.*, how much were they sold for?

Exercise 18

1. **(a)** $\frac{3}{4} \times 1\frac{4}{15} \times \frac{5}{38}$ **(b)** $5\frac{1}{4} \div \frac{7}{12}$ **(c)** $3\frac{3}{8} + 1\frac{1}{12} - 2\frac{5}{6}$

2. Solve and check the following equations.
(a) $3x + 4 = 24 - x$ **(b)** $a - 7 = 3a - 11 + 2a$
(c) $4y + 3y + 2 = 6y + 17$

3. **(a)** $3{\cdot}07 \times 0{\cdot}98$ **(b)** $39{\cdot}382 \div 9{\cdot}7$

4. Town A is 50 m. due N of town B. A third town C is 60 m. from B on a bearing N70°E. Make a careful scale drawing and find, **(a)** the distance from C to A and **(b)** the bearing of C from A.

5. A box measures 1 ft. 3 in. by 8 in. by 6 in. What is its volume in cu. in? If one gross of these boxes is tightly stacked, what will the volume of the stack be in cu. ft.?

6. A father is at present six times as old as his son. In four years' time he will be four times as old as his son. By means of an algebraic equation, find their ages now.

Exercise 19

1. Study the following series of numbers. Find the connection between each number and the one before it. Use this connection to write down in each case the next *two* numbers of the series.
(a) 3, 10, 17, —, —
(b) 9, 27, 81, —, —
(c) 51, 45, 39, —, —
(d) 256, 128, 64, —, —
(e) $3\frac{1}{4}$, $4\frac{1}{2}$, $5\frac{3}{4}$, —, —
(f) 4, 9, 16, —, —

2. **(a)** Add £3 14*s.* 2*d.*, £7 17*s.* 8*d.*, £14 9*s.* 11*d.* and £10 11*s.* 5*d.*
(b) Subtract 3 yd. 2 ft. 11 in. from 7 yd.

3. Two pieces of metal of length 3·47 in. and 2·89 in. are to be cut from a piece 7·8 in. long. What is the length of the remaining piece?

4. If $\frac{7}{16}$ of a number is 217 what is the number? If the number is added to 256 find $\frac{5}{8}$ of the sum.

5. Calculate the angle AOE. Say, with reasons, whether AOD is a straight line or not.

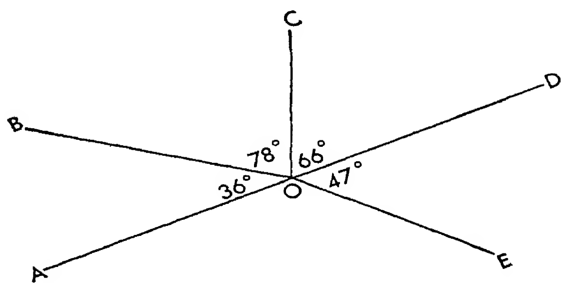


fig. 16.18

6. A man earns 4*s*. 2*d*. per hour and 'time and a half' for overtime. Find in £ *s*. *d*. how much he earns for a week of 44 hours and 5 hours overtime. Out of this money he pays 12*s*. 11*d*. for National Insurance and £1 6*s*. for Income Tax. How much did he actually receive in his pay packet?

Exercise 20

1. (a) Express the following fractions and mixed numbers in decimal form:
   $3\frac{3}{5}$, $\frac{7}{16}$, $1\frac{79}{100}$, $\frac{239}{1000}$, $\frac{763}{10}$, $7\frac{3}{4}$
   (b) Express the following decimals as vulgar fractions in their lowest terms.
   0·375, 0·35, 0·005, 6·0625, 0·364, 0·017.

2. In the formula $S = \frac{n}{2}(a + l)$
   (a) Find S if $n = 16$, $a = 3$ and $l = 12$
   (b) Find l if $S = 350$, $n = 14$ and $a = 4$
   (c) Find a if $S = 150$, $n = 12$ and $l = 24$
   (d) Find n if $S = 132$, $a = 2$ and $l = 31$.

3. A room is 12 ft. long and 8 ft. 9 in. wide. There is a carpet 9 ft. long and 7 ft. 6 in. wide in the centre. What is the area of the border in sq. ft.? How many sq. yd. in this, correct to the nearest sq. yd.

4. A car averages 34·2 miles per gallon of petrol. How much petrol is required for a journey of 461·7 m.? If petrol costs 4*s*. 10*d*. per gallon, find the total cost of the petrol.

5. A bag of potatoes weighs 28 lb. and one-third of its own weight. How heavy is it? (Let the weight of the bag be x lb.)

6. One side of a long-playing record lasts 35 min. and one side of a 'pop' record lasts 4 min. If you had one long-playing record, how many 'pop' records would be required to make up a programme lasting 2 hr. 10 min. if both sides of every record were played? Allow 8 min. of this time for changing the records.

5		
	7	6

fig. 16.19

Exercise 21

1. Copy out and complete the magic square. Each line should total 21.

2. (a) $2\frac{3}{4} + 1\frac{7}{16} - 1\frac{11}{12}$ (b) $7\frac{3}{4} \times 1\frac{17}{31} \times \frac{7}{36}$ (c) $7\frac{7}{8} \div \frac{9}{16}$

3. Find angles x and y.

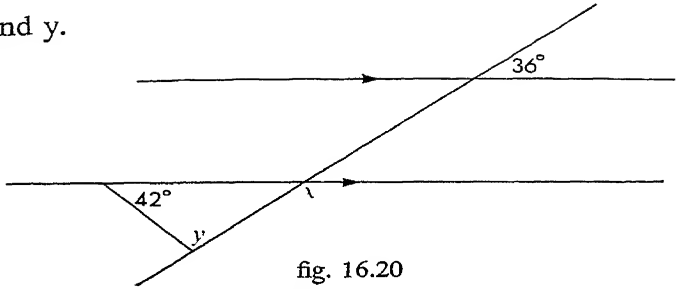


fig. 16.20

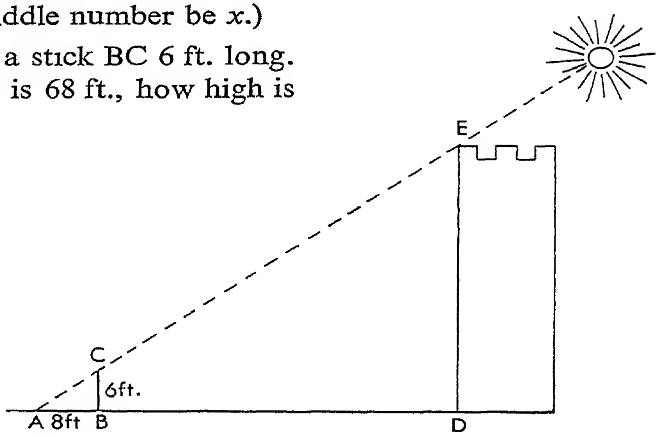
4. If the wages of 6 men in a particular week were £14 11*s.* 8*d.*, £11 0*s.* 10*d.*, £13 5*s.* 7*d.*, £15 0*s.* 11*d.*, £10 19*s.* 4*d.* and £12 4*s.* 8*d.*, find their average wage.

5. (a) Solve and check: $7 + \frac{x}{3} = 2x - 3$.

(b) Three consecutive numbers add up to 63. What are they? (Let the middle number be x.)

6. C is the top of a stick BC 6 ft. long. AB is 8 ft. If AD is 68 ft., how high is the tower DE?

fig. 16 21



Exercise 22

1. (a) What is the product of 14·2 and 8·09?

(b) How many times is 3·72 contained in 76·632?

2. (a) Find the sum of 1 yd. 1 ft. 11 in., 3 yd. 0 ft. $8\frac{1}{2}$ in., 4 yd. 2 ft. $3\frac{1}{2}$ in., 2 ft. $10\frac{1}{2}$ in. and 2 yd. 1 ft. 7 in.

(b) A lorry and its load weigh 12 tons 5 cwt. 1 qtr. If the tare weight (unladen weight) of the lorry is 2 tons 12 cwt. 3 qtr., find the weight of the load.

3. A rectangular field is 75 yd. long and 35 yd. wide. How many yards of fencing are required to enclose it? If a boy walks at 4 m.p.h., how many times can he walk round the edge of the field in half an hour?

4. A trench is 7 yd. long, 10 ft. wide and 4 ft. 6 in. deep. How many cubic yards of earth were removed in digging it? If the volume increases by one-fifth due to the loosening of the earth on being dug, how many lorries each carrying 6 cu. yd. are needed to cart it away?

5. In a particular month a motorist bought 17 gal. of petrol at 4*s*. 6*d*. per gallon. He bought 2 pints of oil at 2*s*. $9\frac{1}{2}$*d*. per pint and paid a bill to the garage for repairs for £2 5*s*. 11*d*. He also paid £1 2*s*. 0*d*. for insurance and £1 5*s*. 0*d*. for road tax. Find the average cost per mile if he did 500 m. (Give your answer in pence expressed as a decimal.)

6. Town Q is 200 m. due W of P. Town R is 150 m. from Q on a bearing N47°W. By means of a scale drawing, find the bearing of R from P, and the distance from P to R. How long would it take an aircraft flying at 150 m.p.h. to fly directly from P to R?

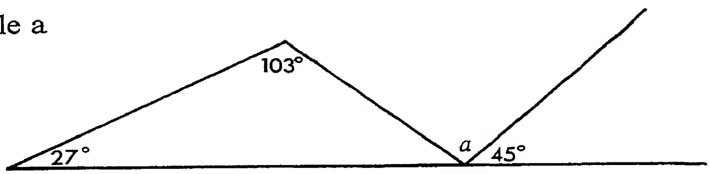
Exercise 23

1. Find:
   (a) $\frac{3}{4}$ of 1*s*. 6*d*.
   (b) 55% of £5.
   (c) 0·375 of 1 ft. (in inches)
   (d) $\frac{17}{40}$ of £16
   (e) $66\frac{2}{3}$% of 5 yd.
   (f) 0·8 of 5·6 in.

2. Solve and check the following equations.
   (a) $12x - 5 = 3x + 13$
   (b) $a + 4 + 3a = 5a - 2$
   (c) $7 - 3s = 9 - 5s$
   (d) $12b + 13b = 400 + 5b$

3. (a) Find angle a

fig. 16 22



(b) Find angles x, y and z

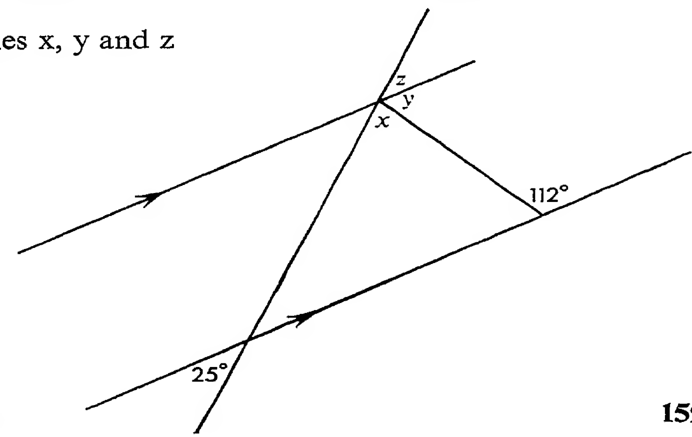


fig 16.23

4. From a bar of metal 6·5 in. long a piece 1·95 in. long is cut off. What length remains? What percentage of the bar has been cut off?

5. A market gardener had 2,000 tulip bulbs to plant out in rows, 27 in each row. If 123 of the bulbs were rotten and could not be planted, how many complete rows did he plant, and how many bulbs were left over?

6. A shopkeeper found at the end of the day that his takings amounted to £4 4*s.* 6*d.* This sum of money was made up of florins and halfcrowns. If there were 38 coins altogether, find, by using an equation, how many of each coin there were.

Exercise 24

1. (a) Add 20·7, 39, 18·03, 0·982 and 361·2
(b) By how much does the above sum fall short of 500?

2. (a) $3\frac{7}{8} + 2\frac{5}{6} - 1\frac{7}{12}$ (b) $3\frac{1}{2} \times \frac{5}{14} \times \frac{16}{55}$ (c) $\frac{17}{38} - 1\frac{15}{19}$

3. At a stamp auction a dealer buys 11 stamps at 2*s.* 6*d.* each, 3 at 5*s.* 0*d.* each, 7 at 7*s.* 6*d.* each and one for £2 3*s.* 6*d.* Find how much he paid altogether and the average cost of one stamp.

4. Find the perimeter and the area of this figure.

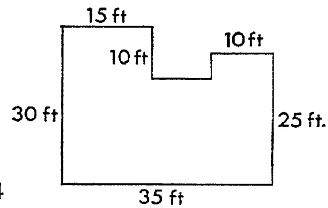


fig. 16.24

5. A man earns £13 10*s.* 0*d.* per week. What are his annual earnings? He is allowed £240 plus $\frac{2}{9}$ of his annual earnings free of income tax. What is the total tax-free allowance? Find how much tax he pays in a year at a rate of 5*s.* 0*d.* in the £.

6. Mary is 5 years older than John and John is 2 years younger than Tom. If their three ages total 34 years, find Mary's age by using algebra.

Exercise 25

1. (a) What is the product of 0, 1, 2, 3 and 4?
(b) What is the sum of 0, 1, 2, 3 and 4?

(c) What is the difference between 4·632 and 5?
(d) By how much does 1,463 exceed 997?
(e) Find the product of 7·07 and 3·09.

2. Express the first quantity as a percentage of the second.

(a) 6*d.* of 2*s.* 6*d.*
(b) 6 in. of 1 yd.
(c) 3 fur. of 1 m.
(d) 4*s.* 6*d.* of £1.
(e) 5 pt. of 3 gal.
(f) 14*s.* 6*d.* of £2 3*s.* 6*d.*

3. Make out a bill for the following items and find the total cost.
2 paint brushes at 6*s.* 9*d.* each.
2 quarts paint at 19*s.* 11*d.* a quart.
2 quarts undercoating at 15*s.* 6*d.* a quart.
16 pieces wallpaper at 9*s.* 0*d.* a piece.
2 pkts. paste at 1*s.* 5*d.* a packet.

4. A father gave £945 to be shared between his three sons. The eldest had $\frac{3}{7}$ and the youngest $\frac{3}{14}$. What fraction of the money did the middle son receive? Calculate the amount received by each son.

5. A circle is divided into 12 equal sectors as shown. Find the following angles: AOB, AOF, reflex COK, DOI.
If OA is due North what is the bearing of:

(a) C from J
(b) K from G
(c) E from I

What fraction of the area of the circle is sector COH?

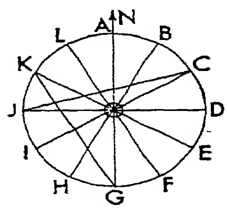


fig. 16.25

6. A man leaves his house at 8.20 a.m. He takes 20 min. to walk to the station and then waits 5 min. for his train. He arrives at his destination, 230 m. away, at 2.30 p.m. Find how long the train took on the journey and its average speed.

Exercise 26

1. Find the missing numbers represented by *'s in:

(a) *Add*

$$\begin{array}{r} 23* \\ *41 \\ 8*9 \\ \hline *824 \\ \hline \end{array}$$

(b) *Subtract*

$$\begin{array}{r} 3**2 \\ 104* \\ \hline *664 \\ \hline \end{array}$$

2. (a) The sum of two numbers is 107·8. One of these is 79·9. What is the other?

(b) The product of two numbers is 23·864. One of them is 7·6. What is the other?

(c) $\dfrac{x}{76{\cdot}3} = 1{\cdot}4$. Find x.

3. If angle ADC = 73°, find x and y.

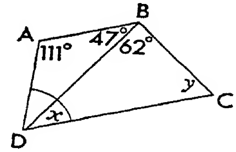


fig. 16.26

4. In a certain church 37% of the congregation are men and 39% are women. There are 72 children present. Find the number present.

5. Solve and check the following equations:

(a) $5r - 7 + 2r = 14 + 3r + 9$

(b) $\dfrac{x}{3} + 21 = \dfrac{5x}{6} - 4$ (c) $\dfrac{x+7}{2} = \dfrac{3x+1}{4}$

6. A man pays £1,376 for a car and a caravan. If the car cost £392 more than the caravan, find the price he paid for each.

Exercise 27

1. How many minutes are there from 10.15 a.m. to 2.10 p.m.?

2. (a) $4\frac{5}{8} - 2\frac{3}{4} + 1\frac{1}{2}$ (b) $3\frac{1}{2} + 1\frac{1}{8} + 2\frac{3}{4} - 5\frac{7}{8}$

3. If the length of a soldier's marching pace is 32 inches, how many paces will he make in marching one mile?

4. When a man sold 7,000 oranges at 7 for two shillings he received what the oranges cost him. If he had sold them at 5 for two shillings, what profit would he have made?

5. In an advertisement it is stated that a car travelled 240 miles in 5 hours at 32 miles a gallon. How many gallons of petrol are consumed each hour?

6. The distance between two wickets was marked out as 22 yards but the yard measure was found to be $\frac{3}{4}$ inch too short. What was the actual distance between the wickets?

Exercise 28

1. (a) How many cwts. are there in $2\frac{1}{4}$ tons?
(b) How many twelfths are equal to one-third?
(c) What fraction is halfway between $\frac{1}{2}$ and $\frac{3}{4}$?

2. (a) A square foot of glass costs 6*d.* What will a square yard cost?
(b) Two pieces of cardboard each 6 in. square are joined to make a rectangle. How long is the perimeter of this rectangle?
(c) Multiply £8 6*s.* 7*d.* by 15.

3. Add together $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$.

4. Solve the equations:
(a) $4x + 5 = 3x + 8$
(b) $\dfrac{x}{4} + 2 = \dfrac{5}{2} - 2x$

5. What number is that to which, if 20 be added, the sum is equal to three times the required number?

6. If in a boat's crew of 8 men the weights of the men are respectively 12, 13, 10, 8, 9, 14, 15 and 11 stones, find the average weight.

Exercise 29

1. Make out the following bill:
5 lb. of potatoes @ 5 lb. per shilling.
3 lb. of butter @ 3*s.* 2*d.* per lb.
$\frac{1}{2}$ lb. of biscuits @ 2*s.* 8*d.* per lb.
$\frac{3}{4}$ lb. of tea @ 1*s.* 10*d.* per quarter lb.

2. What is the angle between the hands of a clock at:
(a) 2 o'clock (b) 8 o'clock (c) 4 o'clock.

3. A garden, in the shape of a rectangle, is 100 yd. long and 33 yd. wide. A path 2 yd. wide is constructed along each fence (inside the garden) and the remainder of the garden is planted. What is the area under cultivation and what is the total area of the path?

4. In the diagram AB = BC = CD = DE = EA. Angle BCD = Angle CDE = 90°
Angle BAE = 60°.
Find the value of the angles ABC and AED. If AB = 2 inches in length, find the perimeter of the figure ABCDE.

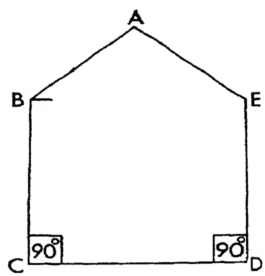


fig. 16.27

5. The average price per cwt. of live cattle at a country market rose during a week from 125*s*. per cwt. to 130*s*. per cwt. Calculate the percentage rate of increase.

6. In a certain borough a rate of 1*d*. in the £ brings in £4,580. If 1*s*. 9*d*. in the £ is the rate allocated to Public Health Services, what is the total cost of these services?

Exercise 30

1. Find the value of:
(a) 852·75 × 0·58 (b) $30\frac{1}{4} \div 5\frac{1}{2}$.

2. A bottle one-quarter full of water weighs $1\frac{1}{2}$ lb. When the bottle is three-quarters full of water it weighs 1 lb. 14 oz. What is the weight of the empty bottle?

3. Express:
 (a) a speed of 75 m.p.h. in yards per minute
 (b) a speed of 88 ft. per second in m.p.h.

4. I sell my car at a loss of 20% for £545. What price did I pay for the car?

5. A swimming bath is 100 ft. long and 30 ft. wide. The average depth of the water in the bath is 5 ft. What is the volume of water in the bath? If 1 cu. ft. of water is equal to $6\frac{1}{4}$ gals., how many gallons of water are there in the bath?

6. Find the angles marked a, b, c, d.

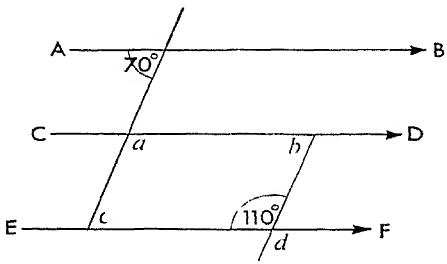


fig. 16.28

Exercise 31

1. The formula for changing the temperature in degrees Fahrenheit to degrees Centigrade is:
"From the given number of degrees Fahrenheit subtract 32 and then multiply the remaining number by 5 and divide the answer by 9." Using this formula, find: (a) 77°F (b) 50°F (c) 122°F (d) 32°F.

2. Using the above formula in reverse, change the following temperatures in degrees Centigrade to degrees Fahrenheit: (a) 15°C (b) 5°C (c) 65°C (d) 100°C.

3. Find the perimeter of the figure drawn in the diagram. Find also the area of the figure.
(Can you spot the easy way of doing the second part of this question?)

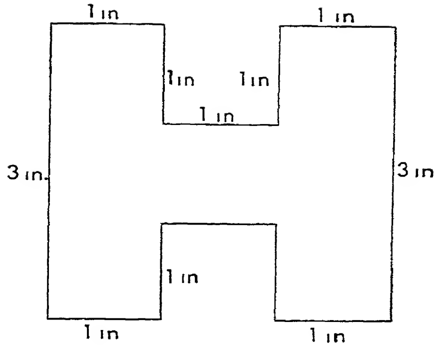


fig 16 29

4. Write down the first twelve natural numbers, i.e., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
Take the first and last and add them together, thus $(1 + 12) = 13$.
Take the second and last but one and add them together, thus $(2 + 11) = 13$.
Take the third and last but two and add them together, thus $(3 + 10) = 13$.
Continue this process and you will get six sets of 13. Thus by multiplying 13 by 6 you get the *sum of the first twelve numbers*, i.e. 78.
Check this result by adding all the numbers together. Use the same process to find the sum of the numbers 2, 7, 12, 17, 22, 27, 32, 37, and check your result by addition.

5. What is:

(a) 5% of £200
(b) the fraction half-way between $\frac{1}{2}$ and $\frac{3}{4}$
(c) $\frac{7}{8}$ expressed as a decimal
(d) the value in shillings and pence of one-third of £1
(e) the difference between $2\frac{7}{8}$ and $1\frac{1}{3}$.

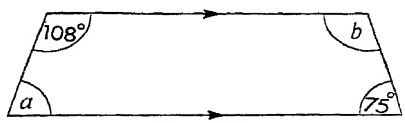


fig. 16.30

6. (a) Find the values of the angles marked a and b.
(b) Town A is N15°E of O and B is N58°E of O.
Find the value of the angle AOB. If OA = OB, what is the value of each of the angles OAB and OBA?

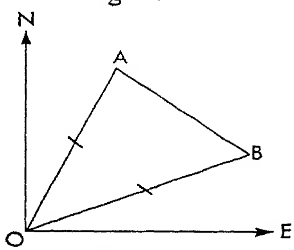


fig. 16.31

Exercise 32

1. In a certain year the Government invested a portion of the National Income in the following proportions:

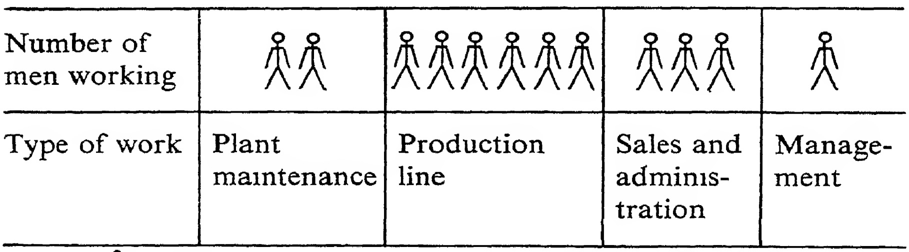
30% in Factories
15% in Mines, Power Stations, etc.
20% in Transport and Communications
5% in Schools and Hospitals
15% in Housing
5% in Farms

and the remaining 10% in miscellaneous enterprises. Construct a pie-chart to exhibit these figures.

2. The average temperatures at mid-day over a certain week were as shown in the following list. Exhibit these figures on a line graph.

Sunday 15° C, Monday 14·8° C, Tuesday 14·1° C, Wednesday 13·5° C, Thursday 14·5° C, Friday 15·2° C, Saturday 15·8° C.

3. Study this diagram:



If each represents 10 men, exhibit the working plan of the factory on a Pie-Chart.

fig. 16 32

4. In a manufacturing town the monthly average of unemployed over 12 months was as follows:

January	550	May	250	September	175
February	525	June	250	October	250
March	450	July	200	November	300
April	400	August	150	December	350

Draw a bar graph to show these figures.

5. A lift in a twenty-storey building makes the following journeys starting at the lowest floor. Up 12 floors, down 4 floors, up 12 floors, down 20 floors, up 10 floors, up 6 floors, down 12 floors, up 8 floors.

Draw a line graph to show these journeys and find at which floor the lift is standing at the end of this period.

6. Oil-Changes in Britain's position since 1938.

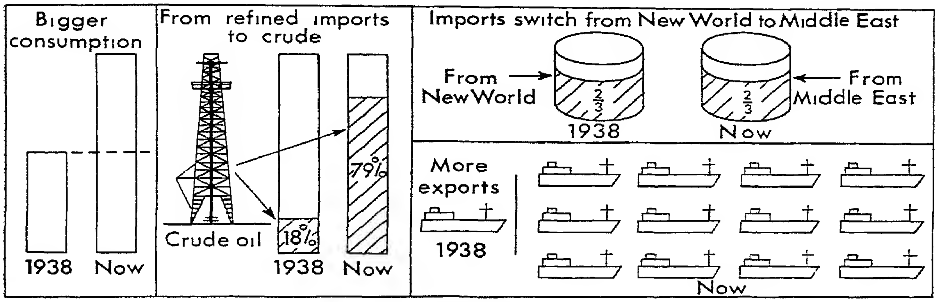


fig. 16.33

Study this diagram carefully and answer the following questions:

(a) What is the percentage increase in imports of crude oil since 1938?

(b) What is the percentage increase in exports of oil products since 1938?

(c) If the present annual value of exports of oil products is £108 m. what was the value in 1938?

Exercise 33

1. Write down the next two numbers in the following sets of numbers (or series):

(a) 1, 4, 7, 11, —, —

(b) $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, —, —

(c) 9, 3, 1, $\frac{1}{3}$, —, —

(d) 1, 1×2, $1 \times 2 \times 3$, —, —.

In part (d) calculate the values of the terms.

2. In a sale, prices were reduced by 10%. What was the sale price of an article whose original price was £9 15*s*. 0*d*.?

3. A workman is paid £7 3*s*. 0*d*. for a 44-hr. week. If in a certain week he has 4 hr. off work, how much should he be paid?

4. (a) What two compass directions are perpendicular to South-East?

(b) A ship is sailing due North and she changes her course 15° towards the West. Later she changes her course 15° towards the East. On what course is she now sailing?

5. Solve the equations:

(a) $8x - 9 = 15 - 4x$

(b) $5y + 3 + 2y = 3y + 27 - 4y$.

6. Express 1 cu. ft. in cu. in. If the size of a normal building brick is 9 in. long, 3 in. wide and 2 in. thick, how many bricks will be required to make a volume equal to 1 cu. ft.?

Exercise 34

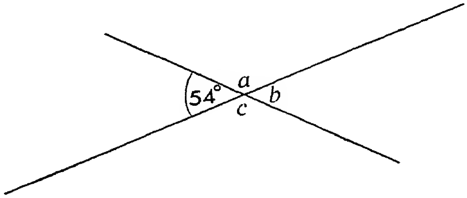


fig. 16.34

1. Find the angles marked a, b, c.

2. Look at this extract from a railway timetable:

Waterloo	4.50	5.20	6.5
Woking	—	—	6.35
Guildford	5.25	5.57	6.47
Portsmouth	6.27	7.9	8.0

(a) Why is there no time stated at Woking for the trains which leave Waterloo at 4.50 and 5.20?

(b) If the train which leaves Waterloo at 6.5 is 25 min. late arriving at Guildford and loses 10 more min. on the journey between Guildford and Portsmouth, what time will it arrive at Portsmouth?

3. Draw a triangle whose sides are respectively 3 in., 4 in. and 5 in. long and another triangle whose sides are respectively 5 cm., 12 cm. and 13 cm. long. Measure the angles in each of these triangles and see if you can discover one angle which has the same value in both triangles. What is the size of this angle in degrees?

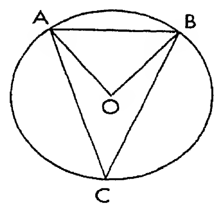


fig. 16.35

4. Copy this diagram in your books (Radius of circle 1 in.).
Measure the angles AOB and ACB using your protractors.
What do you discover about the angles AOB and ACB?

5. A staircase has 15 steps. If the rise (height) of each step is 9 in. and the tread (width) of each step is 12 in., how far has a man travelled (a) vertically (b) horizontally in climbing these stairs? Give your answers in feet.

6. If AB = AC in length, find the values of the angles marked a, b, c, d.

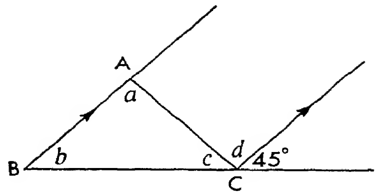


fig 16.36

Exercise 35

1. A concert hall has 400 seats. When a concert is organised a certain number of tickets are sold at 5*s.* each and the remainder at 4*s.* each. If the total amount received is £89, how many 5*s.* tickets were sold?

2. Express £2 5*s.* 0*d.* as a fraction of £7 10*s.* 0*d.* in its lowest terms.

3. Copy this diagram in your book. (Radius of circle 1 inch.)
Measure the angles marked a, b, c, d.
Find the values of a + c and b + d.

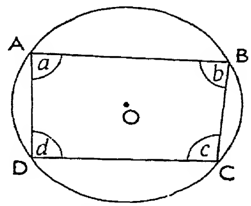


fig. 16.37

4. Draw a circle, radius 1 in.; using the same radius, measure and mark equal arcs along the circumference of this circle. Join the arcs, thereby constructing a regular hexagon (see diagram). Join the corners of the hexagon to the centre of the circle and measure the angles of the triangle so formed. Use this information to find the total of the six angles at the corners of the hexagon.

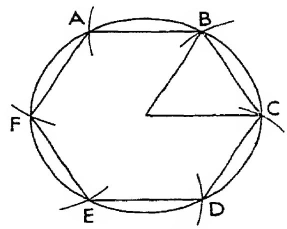


fig. 16.38

5. The diagram represents a balance. If the arm is to balance, then the weight in the left-hand scale pan by the distance to the balancing point must be equal to the weight in the right-hand scale pan by the distance to the balancing point.
i.e. ax = by. Find the value of
(a) if x = 4, y = 7, b = 8 and
(b) if x = 10, y = 5, a = 9.

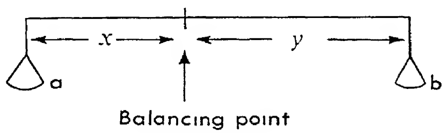


fig. 16.39

6. Express a speed of:
- **(a)** 60 m.p.h. in ft. per second.
- **(b)** 44 ft. per sec. in m.p.h.

Exercise 36

1. What will it cost a motorist for petrol and oil for a journey of 200 miles if he uses 1 gallon of petrol for every 32 miles and 1 quart of oil for every 1,000 miles?
Petrol costs 4*s.* 6*d.* per gallon and oil 15*s.* per gallon.

2. Find the missing angles in the following diagrams:

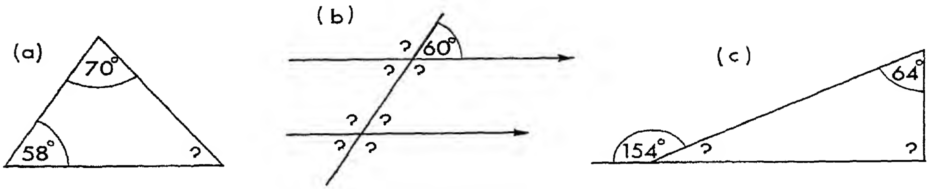


fig. 16.40

3. Solve the equations:
 (a) $8x - 16 = 56 - 4x$
 (b) $2y + 16 + 5y = 11y$.

4. A watch which loses 8 seconds every hour is set to correct time on Monday at noon; what will be the time shown by this watch at noon on the following Saturday?

5. Cambridge is 16 miles due east (090°) from St Neots and Huntingdon is 8 miles from St Neots in a direction 025° (N25°E). Using a scale 1 inch = 4 miles, draw a diagram showing the relative positions of Cambridge, St Neots and Huntingdon and find the distance from Cambridge to Huntingdon and the bearing of Huntingdon from Cambridge.

6. A boy's marks in four examination papers were 33 out of 50 marks, 41 out of 60 marks, 14 out of 25 marks and 24 out of 40 marks. Express the sum of the marks gained by the boy as a percentage of the total obtainable.